MANPOWER PLANNING IN HIERARCHICAL ORGANISATIONS:

A MIXED INTEGER PROGRAMMING APPROACH

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DECLARATION

I declare that this thesis has been composed by me and that the work is my own.

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ABSTRACT

Manpower planning is concerned with planning the use of human resources. In this thesis, manpower planning is defined as the process of determining manpower policies which ensure that suitable numbers of qualified people are in appropriate positions at the right times in order to meet organisational goals, while taking account of the career development opportunities of the individuals within the organisation.

A number of different mathematical models have been developed for manpower planning. These models are reviewed and it is noted that a weakness of the optimisation models which have been proposed is that promotion rates, i.e. the proportion of staff promoted per year, can vary substantially from year to year because of the limitations of the techniques used. Since staff morale is likely to be affected if promotion rates vary significantly from one year to another, the results from these models may be unacceptable to management. In this thesis a mixed integer programming (MIP) manpower planning model is developed for determining minimum cost manpower policies in which promotion rates remain stable over time, and which satisfy specified staffing level requirements. In this MIP model promotion rates are treated as decision variables by using a binary variable representation. An iterative procedure is developed for solving this MIP model.

The computational aspects of using the MIP manpower planning model are investigated. A demonstration decision support system based on this MIP model is developed, and the use of this system is illustrated using

representative data for a military manpower system. The experience with this demonstration system suggests that the approach could be developed to produce a practical tool to aid management decision making.

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INTRODUCTION

1.1 MANPOWER PLANNING

Manpower planning is concerned with planning the use of human resources. The term manpower planning dates, as noted by Smith and Bartholomew (1988), from no earlier than 1960. Although the term is relatively new, the problems of manpower planning may have existed many centuries ago. It must have been examined by those responsible for building the Great Wall in ancient China or the pyramids in ancient Egypt, while Jones (1964) had ascertained that research on the career structures, wastage rates and promotion prospects in the Royal Marines had been embarked upon as early as 1779. Perhaps because of its long history, manpower planning is not easy to define. Everyone has his or her own point of view. Some views on manpower planning will be discussed in this chapter, and the objective of this research will be outlined.

1.1.1 The Meaning of Manpower Planning

Some definitions of manpower planning can be very general. For example, the Department of Employment (1974) defined manpower planning as a strategy for the acquisition, utilisation, improvement and retention of an enterprise's human resources. In other cases more details of the nature and objectives of manpower planning are given. Smith (1974) described manpower planning as an approach to the management of human resources, but noted that since any imbalance between personnel and other resources is likely to involve unnecessary expenditure of one kind or another, the general aim is to reduce the risk of either surpluses or shortages of particular kinds of manpower. Bowey (1974) regarded manpower planning as the activity of management which is aimed at coordinating the requirements for, and the availability of, different types of employee. This coordination is needed to ensure that the organisation has enough of the right kind of labour at the time it is required, but it may also involve adjusting the requirements to the available supply.

Grinold and Marshall (1977) suggested that ideally the goal of manpower planning within an organisation is to ensure that the correct numbers of the right types of people are in the appropriate jobs at all times, but in practice a more realistic goal is to avoid having too many of the wrong types of people in the inappropriate jobs too frequently. Vajda (1978) viewed manpower planning as arranging for the right number of individuals to be allocated to various well-defined activities. Charnes, Cooper, and Niehaus (1978) considered that manpower planning should include forecasting both the demand and supply for specific skills in all the activities associated with personnel recruitment, assignment, training, promotion and transfer. Bartholomew and Forbes (1981) defined manpower planning as the attempt to match the supply of people with the jobs available for them.

Most definitions of manpower planning focus on organisational interests. Only some definitions explicitly consider both the organisation and the individuals within it. For example, although Gohl and Opelland (1978) considered manpower planning to be an integrated part of corporate planning concerned with ensuring a supply of people with the necessary qualifications, they also acknowledged the importance of recognising the goals of individuals. Verhoeven (1982) viewed manpower planning as a set of activities designed to maintain the required numbers of employees with the necessary qualifications in order to realise the organisation's goals while taking into consideration the interests of individual employees concerning career development opportunities, salary levels, working conditions, protection from dismissal or involuntary transfer, etc.

From the above it can be seen that manpower planning is generally considered to be an approach or set of activities related to the management of human resources, and that the main purpose of manpower planning is to coordinate manpower demand and supply. Most definitions focus on organisational interests, but the need to consider the individuals within an organisation is also recognised. In our view, manpower planning should not only consider organisational goals but should also take account of the needs and aspirations of individuals within the organisation, since most activities in the organisation must be carried out by its employees.

Although the ultimate goals of organisations are diverse, concerning profit, productivity, service, national security etc., there is little prospect of realising the organisational goals if the whole-hearted support of the individuals is lacking. This research is concerned with the manpower related goals of the organisation in the context of the ultimate goals of the entire organisation. Since these manpower goals must be consistent with the overall organisational goals, the manpower related goals are normally focused on matching the demand for and supply of manpower, both qualitatively and quantitatively, and minimising manpower costs. Other factors, e.g. working conditions, salary levels, protection from dismissal or involuntary transfer, which are of concern to the individual can affect the morale of employees. Many of these factors can be considered as part of the day-to-day responsibility of personnel management, while others must be considered as part of organisational strategy. For example, for the maintenance of employee morale it is desirable that the career prospects of individuals, e.g. the likelihood of promotion and the expected waiting time before promotion, do not depend on when they entered the organisation. This research will concentrate on the structural aspects of manpower policy, in terms of the number of people in each grade and the transition rates between grades. This approach is adopted in an attempt to provide a framework within which the career development opportunities of individuals can be viewed.

In this research, manpower planning is defined as the process of determining manpower policies which ensure that suitable numbers of qualified people are in the appropriate positions at the right times in order to meet organisational goals, while taking account of the career development opportunities of the individuals within the organisation. The achievement of organisational goals will involve matching the demand for, and supply of, manpower and minimising manpower costs, while it is assumed that career development opportunities, in terms of measures such as the probability of promotion and the expected waiting time before promotion, should remain relatively stable over time. Manpower planning is therefore regarded as a process consisting of a series of stages associated with the management of human resources. This definition is similar to that of Grinold and Marshall, but with an emphasis on both organisational goals and the career development opportunities of individuals.

In this definition, suitable numbers suggests the quantitative matching of the demand for, and the supply of, manpower. The emphasis on qualified people in appropriate positions reflects both the qualitative matching of the demand for, and the supply of, manpower, and the consideration of the career development opportunities of individuals. In examining the qualifications of individuals, not only education, training, experience, performance, service-length etc., but also the suitability of individuals for particular jobs should be considered. For example, a job in which there is much contact with the public may not be suitable for those who are of a retiring disposition even though they appear to have appropriate formal qualifications. The consideration of appropriate positions not only refers to the tasks or jobs to be performed, but also relates to the posts to which individuals should be allocated. Timing must be considered not only because of the need to supply manpower in time to meet the requirements of the organisation, but also because of the need to promote in time to satisfy the aspirations of individuals.

1.2 THE NEED FOR MANPOWER PLANNING

It has been noted that the main purpose of manpower planning is to harmonise the demand for, and the supply of, manpower while taking account of the career development opportunities of individuals. If the manpower demand and the manpower supply have been constant over a long time, the matching of the demand and supply can be executed easily, and manpower planning becomes a routine process. However, stable organisational structures of this form are not common in practice. This implies that manpower planning is always needed. Manpower planning can also provide early warning of potential problems arising from proposed manpower policies. Because the manpower policies of an organisation have an impact on both the organisation and the individuals, manpower planning is essential to the efficient operation of all organisations.

1.2.1 The Stability of Organisations

Stable organisations are not common in practice since environments are dynamic and organisations are influenced by environmental factors. These factors include the influence of competitors, changes in technology, the state of the labour market, the power of trade unions, government policies, economic factors etc. For example, innovation may lead to new methods of production which require more skilled employees or training the current employees to adapt to these changes. Organisations which do not adapt to change will ultimately die.

In a dynamic environment, manpower requirements and wastage will vary. Thus future manpower demand and supply should be evaluated, and policies must be analysed in order to reduce the possibility of discrepancies between demand and supply. The full effects on organisations and individuals of manpower policies relating to recruitment, promotion, retirement, etc., cannot be identified immediately. The impact of today's decisions may be disclosed a long term later. Therefore, these policies must be analysed before implementation to avoid causing future problems which cannot be remedied easily. Manpower planning is an essential part of strategic policy formulation.

Since organisations are constantly changing, the age structure of staff and salary and pension costs will fluctuate. Bulges in the age distribution of staff can lead to unequal career development opportunities for those who enter the organisations at different times. People who have poor prospects and better qualifications may leave. Some people may become so frustrated as to influence the quality of their work and that of other employees around them. If there are no proper policies developed in order to obtain a better age distribution, an irregular age structure will perpetuate for a long period. Salary and pension costs will be influenced by the number and age distribution of employees in different grades. Fluctuating costs can result in difficulty in operating the organisation. Manpower planning should reveal such potential problems and enable remedies to be evaluated.

1.2.2 The Role of Manpower Planning

Different combinations of manpower policies can lead to quite disparate results in the supply of manpower and the career opportunities of individuals. Because of the complexity of the combinations, it is unlikely that the full consequences of manpower policies will be realised without conducting manpower planning in a formal manner. For example, a change in retirement age can lead to changes in the number of people in each grade, the promotion opportunities of individuals, and the pension costs. These changes in retirement age and promotion opportunities can have long-term consequences. The knowledge of alternative manpower policies which produce similar results is useful because it gives management the opportunity to choose policies that best suit their situation. By using formal manpower planning techniques these policies can be evaluated more efficiently, and the consequences of different manpower policies can be investigated before decisions are made.

1.3 THE PROCESS OF MANPOWER PLANNING

The process of manpower planning is frequently described (e.g. Edwards, 1986) as consisting of three stages:

1. forecasting the demand for manpower;
2. forecasting the supply of manpower;
3. matching the demand for, and the supply of, manpower.

Although this is generally a clear and convenient way to describe the process of manpower planning, it can misrepresent the process in two ways. Firstly, it seems to suggest that the process of manpower planning is a series of sequential stages which are carried out once and then left. Secondly, it appears to neglect the influence of the environment on manpower planning. Clearly, the process of manpower planning is continuing and never ending since the environment is dynamic and the organisation and its staff are influenced by them. The interaction between the environment and the stages in manpower planning process is now considered.

A more detailed description of the process of manpower planning can be obtained by considering this process as consisting of six interacting stages.

1. Gather information on the environment, the organisation and the individuals within the organisation.
2. Forecast the manpower supply.
3. Forecast the manpower requirements of the organisation.

4. Establish the range of policies to be evaluated.
5. Establish criteria for evaluating manpower policies, considering both the organisational goals and the career opportunities of individuals.
6. Evaluate the policies in terms of the criteria.

This process is not necessarily serial, and some back-tracking may be required. For example, the nature of some of the manpower policies may affect the supply of manpower.

Effective manpower planning requires a sound information base which can reflect changes in the environment, the organisation, and individuals as soon and as accurately as possible. Therefore, the first stage of the process of manpower planning is to gather information. Relevant information includes government policies, economic indicators, changes in technology, the ability of competitors, the state of the labour market, the attitude of trade unions, organisational plans, possible manpower policies, the current manpower distribution, wastage rates, and the opinions of individuals on career opportunities.

The information gathering in stage 1 provides a base for forecasting manpower supply and requirement of the organisation, and establishing the criteria for evaluating manpower policies. The manpower supply of the organisation in subsequent years depends on the current manpower distribution, the wastage rates, and the possible manpower policies, e.g. promotion and recruitment policies. The manpower requirements of the organisation are affected by factors such as changes in technology, consumer attitudes, economic factors, and the goals of the organisation, and it may therefore be necessary to use subjective methods in forecasting manpower requirements. The criteria for evaluating manpower policies should not only consider organisational goals but should also take account of the career prospects of the individuals within the organisation. Therefore, manpower costs, the stability of promotion rates, the probability of eventual promotion to a specified grade and the expected waiting time to reach that grade, should be considered in attempting to match manpower supply and demand.

If any manpower policies are unacceptable in terms of the criteria adopted, for example the supply is less than the requirements or the supply matches the requirements but the expected waiting time before promotion is unsatisfactory to the decision makers or individuals, at least one of the following actions should be carried out:

(a) develop alternative manpower policies

(b) revise the manpower requirements

(c) revise the criteria

and the process continued. Note that the adoption of any of the actions should be based on relevant information. Since the environment is dynamic, information must be updated in order to adapt to changes in the environment. The process of manpower planning is therefore cyclic and never ending.

1.4 THE OBJECTIVE OF THIS RESEARCH

The objective of this research is to develop a manpower planning decision support system for a hierarchical organisation. The system will be designed to allow minimum cost recruitment, retirement and promotion policies which satisfy manpower requirements, and maintain stable promotion rates over time, to be evaluated.

It has been noted that manpower planning should not only consider organisational goals but should also take account of the career development opportunities of individuals. In particular, stable promotion rates over time help demonstrate to individuals that their career prospects do not depend on the time at which they entered the organisation. The proposed manpower planning decision support system will be based on a new mathematical programming manpower planning model which allows promotion rates, defined in terms of the proportion of staff promoted per year, to be regarded as decision variables. It is suggested that the career prospects of individuals can be considered in terms of the probabilities of eventual promotion to a specified grade, and the expected waiting time to reach that grade. It is shown that these measures can be derived from the manpower planning model on which the manpower planning decision support system is based.

Since the environment is dynamic and organisations are influenced by their environment, manpower policies must adapt to the changes in the environment. The decision support system is therefore required to be able to evaluate policy in a changing environment. The amount of information produced in numerical form by the decision support system may make it difficult to identify the important points in the output. In order to ease the interpretation of the output, graphical presentation will be used in the decision support system.

CHAPTER 2

MODELS IN MANPOWER PLANNING

2.1 INTRODUCTION

A manpower system consists of people who are classified in some suitable way, e.g. in terms of age, length of service, or grade within a hierarchical structure. The manpower system can be described in terms of the number of people in each class, the flows of people between classes and the numbers entering and leaving the system. In a hierarchical system the flows between classes represent promotions and demotions.

There has been considerable work on the development and use of manpower planning models over the last three decades. Most of the published work in this area concentrates on modelling the supply of manpower and little is devoted to forecasting manpower requirements. This reflects the complexity of the demand process which involves factors, such as new technology, competitors' activities, consumer attitudes, market share, and the economic climate. In practice the manpower demand forecast is generally based on planned developments within the organisation.

In contrast, there are many manpower supply models. These models can be categorised according to the approach adopted, the main types of model being Markov models, renewal models, renewal models containing Markov flows, career stream models, optimisation models, simulation models and models based on system dynamics. These models, together with a small number which do not fit into these categories, are discussed below.

2.2 MARKOV MODELS

In Markov models, the number of people in each class is variable. The future class of an individual depends only on his or her immediately preceding class, i.e. is not dependent on the process involved in reaching this class, and the transition rates are stationary, i.e. they do not change with the passage of time. In the models people are transferred, i.e. promoted, demoted or remained, in a fixed proportion independent of the vacancies occurring in each class. The objective of these models is to forecast the number of people in each class. These models have been described in Bartholomew and Forbes (1981), and Bartholomew (1982).

Markov models can be used in a number of ways, e.g. Verhoeven (1982) and Venema (1988) using a Markov model framework developed an interactive manpower planning system, FORMASY. This system has been applied in the Royal Netherlands Air Force and can be used in:

- forecasting the manpower distribution in each class, the steady state distribution of staff, the age distribution of staff, salary costs and the statistical errors in these forecasts;
- providing career information such as average grade seniority on promotion;
- examining the consequences of changing promotion, recruitment, and retirement policies.

The FORMASY system contains many options to facilitate changing the transition matrix, numbers recruited, and retirement age. In this system, manpower policies, e.g. promotion and recruitment rates, can be varied until the computed availabilities agree with a specified requirement in each grade. In this case, it may require much time and computational effort to find suitable policies.

There are many other examples of Markov models in manpower planning. For example, Forbes (1971) used goodness of fit analysis to test the transition probability in a Markov model. Woodward (1983) has allowed three variables - grade (defined by salary bands or work functions), age and length of service - to be used in categorisation of staff. Davies (1973, 1975, 1976) has considered the attainability and maintainability of the grade structure in a manpower system. Bartholomew (1977) has, from a stochastic rather than deterministic point of view, considered the maintainability of the grade structure. Glen (1977) stressed the importance of maintaining a stable grade structure of suitably qualified staff in evaluating manpower policies, and demonstrated that the experience of staff in future staffing structures can be considered in terms of the total length of service distributions. Alam (1984,1985a,1985b) has developed a method based on Markov manpower models for evaluating steady state career structures and seniority distributions for officers in the Indian Air Force and Indian Army.

A crucial weakness of Markov models is that they permit the number of staff in each class to fluctuate too much. In many organisations the number of staff in each class is constrained by the amount of tasks to be done or finance available. When promotion and recruitment can only take place to fill vacancies as they occur, this will lead to difficulties in implementing the policies suggested by Markov models.

2.3 RENEWAL MODELS

In renewal models, the number of people in each class is fixed. The rates of promotion in each class are unknown and fluctuate. People are promoted or recruited only when vacancies occur in a higher class. If vacancies due to leaving and promotion occur in a class, they may be filled either by direct recruitment from outside or by promotion from the class below. The objective of these models is to forecast the number of promotions and recruits in order to match the desired number of people in each class. This can be achieved by filling the vacancies as they occur. Renewal models have been described in Bartholomew (1963, 1982), and Bartholomew and Forbes (1981).

Renewal models have been applied in many ways, particularly in organisations with strict manpower requirements. For example, Boller *et al.* (1978) using a renewal theory approach designed a manpower planning model, FAST, which has been used by the US Navy to develop recruitment, training, promotion and budgetary plans for some 500,000 enlisted personnel. The FAST model has also been used in testing the feasibility of future manpower requirements. In this model staff are classified in terms of 100 occupational specialties, 31 total length of service intervals, and 9 pay-levels. Manpower requirements are issued by the Navy periodically. These requirements are based on a variety of factors, concerned with current and prospective missions, size and configuration of the fleet, numbers and types of weapons and support systems, ship construction schedules and decommissioning plans, needs of the shore establishment, e.g. hospitals, shipyards and repair facilities, and deployment and training schedules. The goal of this model is to develop manpower policies in order to achieve these manpower requirements.

There are many other examples of renewal models in manpower planning. For example, Butler (1974) has presented an approach to equalise the promotion rate between grades within departments with similar hierarchies. In this model, recruitment only occurs in the lowest grade and vacancies are filled by promotion from the immediately lower grade. The equalising of promotion opportunities is achieved by means of a pooling system which allocates promotees between departments. Robinson (1974) has developed a two-stage renewal model in which stage 1 corresponds to the training grade and stage 2 to the organisational grade in which the hierarchical grades are treated as one grade. In this model, the number of people in the training grade can be determined in order that people spend some predetermined length of service in this grade before promotion.

A major limitation of renewal models is that promotion and recruitment are determined only by filling vacancies so that promotion rates can vary significantly from year to year. Therefore, renewal models are not very appropriate in evaluating the impact of career development policies, e.g. policies for equalising promotion opportunities for individuals.

2.4 RENEWAL MODELS CONTAINING MARKOV FLOWS

In renewal models containing Markov flows, the manpower requirement in each class is fixed, and people are promoted or recruited only when vacancies occur. The vacancies in a class can be filled by transfer from any other class rather than only from the class below as in renewal models, or by recruiting from outside. When using these models, the following steps should be followed:

1. evaluate the number of leavers in each class;
2. consider the number of proposed promotions and demotions from each class;
3. calculate the number of vacancies in each class, i.e. the sum of the number of leaving a class and the number of proposed promotions and demotions from the class;
4. fill the vacancies in the top class by considering the number of proposed promotions to this class first, then the remaining vacancies are filled by recruitment from outside or promotion from the second top class; in this case, new vacancies in the second top class are created;
5. add the new vacancies to the vacancies in the second top class and continue the filling process as in the top class, then throughout the system until all vacancies are eliminated.

The objective of these models is to forecast the number of recruits and promotions in order to match the manpower requirements while ensuring flexibility in manpower promotion and demotion policies.

Forbes (1974) and Wishart (1976a) constructed KENT, a manpower planning model of this form. In this model, the grade sizes are predetermined by demand assumptions. The vacancies caused by retirement and wastage in a grade are filled by either recruitment from outside, or promotion or demotion from other grades. The numbers of recruits, promotions and demotions in a grade are determined by applying a vacancy transition matrix, in which the element in the ith row and jth column is the proportion of vacancies in grade i filled by grade j. All the flows out of a grade, i.e. the number of proposed promotions and demotions from this grade obtained by applying the vacancy transition matrix, must be determined before considering flows into this grade. A fluid grading system, in which the joint size of the grades is fixed but the number of people in individual grades is flexible, is used in this model, together with four types of promotion. The type of promotion which is chosen depends on the kind of flow in the system, e.g. promotion may occur only from the grade below or can be from any other grade, and whether the emphasis lies with the requirement at the upper grade or promotion rate at the lower grade. The fluid grading policy is normally adopted when people in the lower grades who reach a certain seniority are presumed qualified for promotion and are promoted irrespective of whether there are vacancies in the higher grade. The promotion profile in the KENT model is based on age rather than seniority in grade. Therefore, as noted by Jones (1978), the output in the form of promotion rates does not provide information about waiting time before promotion.

Manpower planning models of this type have also been developed and implemented on personal computers. For example, Billington (1987) has constructed a Managerial Manpower Model which was applied in Barclays Bank. Pictorial output and user friendliness are the essential characteristics of this model. Spreadsheets have also been used to model manpower flows, e.g. Malloch (1986, 1988) used LOTUS, and Anthony and Wilson (1990) adopted SuperCalc. These flows can be push flows (Markov flows) or pull flows (renewal flows).

Renewal models containing Markov flows are similar to renewal models but with more flexible promotion policies. However, as in the renewal models, both of these type of model concentrate on organisational goals, e.g. sufficient manpower, but pay little attention to the goals of individuals.

2.5 CAREER STREAM MODELS

In career stream models, the number of people in each class is not specified but the total number of people in the manpower system is fixed. Instead of using a transition matrix as in Markov models, people are promoted in accordance with their age and career stream, defined by the grade eventually reached. In these models, e.g. Morgan (1979), the number of staff in each class is determined using an age distribution projecting equation and a career streams diagram, in which the proportion of staff in each career stream and the typical promotion age from each grade are specified.

Keenay, Morgan, and Ray (1977a) developed a model of this form to measure the career prospects, in terms of the proportion of staff eventually promoted to a specified grade, and to forecast the number of staff in each grade within a hierarchical organisation. The initial career prospects, which are represented using a career streams diagram (see diagram 1), are estimated from the staff of the organisation by looking at the proportions of staff in each grade at each age. On this diagram, the horizontal and vertical axes denote age and proportion of staff in or below a grade respectively. At each age, the cumulative proportions of staff in each grade are denoted by points on a graph. Points of the same grade at each age are connected, and the boundary curves for promotions to next higher grade in terms of age are constructed. These curves can be smoothed by hand or estimated directly by using the logistic equation:

$$y_i(x) = q/(1+\exp(k(A-x)))$$

where

$y_i(x)$ is the proportion of those aged x who are in or below grade i

q is the proportion of staff who retire above grade i

A is typical age of promotion to grade i

k is inversely related to the spread of promotion ages around A

q, A and k are parameters which must be estimated from data.

The asymptotes of the logistic curves are extended back to the start of the age range, thus dividing the staff into career streams. The proportion of staff in a specified career stream is the width of the stream, which can be read from the vertical axis of the diagram. Each typical promotion age from one grade to a higher grade in the stream could be estimated by eye from the curves intercepted by the horizontal lines, i.e. the asymptotes. Having constructed the career streams diagram, the next step is to project the age distribution forward. This can be achieved by forecasting the future size of the manpower system, the number of leavers at each age, and the age distribution of recruits. Once the career streams diagram is known and age distribution can be projected, the grade sizes can be forecast by assuming that the diagram remains constant over time and assuming that all promotions occur at the typical age of promotion for each stream. The effect of changing the typical age can then be examined. Origins and extensions of models of this type can be found in Morgan (1971, 1979), Keenay (1974), Keenay, Morgan, and Ray (1974, 1977b), Keenay and Morgan (1979), and Ray (1977).

In career stream models, career prospects and the age distribution of staff together constitute a framework for describing the manpower system. The advantage of these models is that from this simple framework, the number of staff in each grade can be forecast and the effect of changing promotion policies, e.g. typical promotion age, can be examined easily. In these models, age is considered in the promotion policy. By giving different typical promotion ages to the grades in each career stream, the effect on the system can be observed. However, in practice it is unlikely that this approach can be adopted because it is difficult to identify which career stream individuals belong to before they leave the system. Therefore, as noted by Bartholomew and Forbes (1981), these models should be regarded as descriptive rather than predictive models.

2.6 OPTIMISATION MODELS

Optimisation models for manpower planning are concerned with optimising an objective function subject to a set of constraints. The objective function in these models can be cost, manning levels, deviations (both over and under achievement) from target value, and others. The constraints in these models can be concerned with numbers promoted, recruitment levels, manpower requirements, manpower stocks, and finance. These models seek to determine manpower policies which optimise the objective function under these constraints. The optimisation approaches used in manpower planning can be distinguished as linear programming, goal programming, interactive augmented weighted Tchebycheff method, and dynamic programming.

2.6.1 Linear Programming Models

Linear programming (LP) models involve a linear objective function and set of linear constraints. A number of manpower planning models of this type have been developed and implemented. For example, Morgan (1970) developed a cost minimisation LP model for the investigation of manpower policy in the UK Royal Air Force. In this model the decision variables were the number of staff, the number of promotees, the number of redundancies, the number of recruits, and the number of re-employments. During their first twelve years of service, staff can be re-employed up to the twelve year point at their own request; after twelve years re-employment is not automatic and depends on the requirements of the force. When personnel are employed beyond the twelve year point, they become eligible for pension, the pension increasing with every year of service. The total cost used as the objective function of this model included the cost of recruiting, the cost of redundancy, the cost of over-staffing, and pension costs. The constraints in the model related to basic manpower requirements, restrictions on the number of recruits, redundancies and re-engagements, and upper and lower bounds on the proportion promoted to each rank in each age band. A major difficulty with this model lies in determining realistic costs for the objective function. For example, determining the cost of overstaffing seems to be an art rather than a science, and is likely to be a cause of argument.

Because of the considerable uncertainty surrounding the estimation of the various parameters incorporated into an objective function, Purkiss (1970) considered a hierarchy of objectives and investigated the solution at each level of the hierarchy before proceeding further. He developed a linear programming model for the Iron and Steel Industry Training Board in UK. Two numerical examples of minimising redundancies and minimising costs were presented. The dilemma between minimising redundancies, which causes higher costs, and minimising costs, which creates higher redundancies, would then be left to management to decide. Vajda (1978) has also developed a linear programming model to determine a manpower recruitment policy in order to minimise manpower stock cost and recruitment cost while ensuring the total manpower requirement is constant over time.

In LP models, the numbers promoted from each grade in each year are decision variables. Unless there are also restrictions on the proportions of staff promoted per year from each grade, there may be substantial variation in promotion rates from year to year. Although it is clearly desirable to incorporate constraints of this form, these restrictions on promotion rates must be specified in advance, i.e. promotion rate is a parameter in these models. It would, however, be better if promotion rates were regarded as decision variables, but since numbers promoted is the product of promotion rate and number in grade, both of which are variables, the model would be non-linear.

2.6.2 Goal Programming Models

Instead of using only one goal or objective as in linear programming models, multiple goals, e.g. target for promotion and total manpower requirements, are considered in goal programming models. These goals, i.e. target levels, can be over or under achieved. The objective of these models is to minimise deviations from these goals subject to various constraint configurations.

Gass *et al.* (1988) developed a model, MLRPS, of this type to project the strength and cost of the active U.S. Army for 20 years. The model is used by the Army Office of the Deputy Chief of Staff for Personnel in long-range planning, policy planning and force structure analysis, and cost analysis. This model is divided into three major subsystem, the data processing subsystem, the flow model subsystem, and the optimisation model subsystem, with output from one subsystem used as input to the next subsystem.

The data processing subsystem performs three tasks: (1) it collects and stores data from external Army sources, (2) it uses these data to generate historical rates, and (3) it allows the user to generate projected rates such as promotions and skill migrations rates. The subsystem produces output files that are used as input to the Flow Model and Optimisation Model subsystems.

The flow model subsystem uses a Markov chain model to project the flow of the initial force (determined in the data processing subsystem) to a future force over a 20-year horizon, and to generate annual costs of manpower, which include recruitment, training, re-enlistment, wastage, retirement, and pay and allowances. The output of the flow model tells the policy analyst what the future force structure will be, not what it should be.

The optimisation model subsystem uses goal programming to analyse how the Army can best achieve a desired future force structure, starting with its present manpower inventory. This subsystem includes six kinds of constraints, namely manpower stocks, recruitment bounds, wastage goals, promotion goals, grade target goals, and total force target goals. The objective is to minimise the weighted sums of the under- or over- achievement of the wastage, promotion, grade, and total force targets.

The MLRPS model provides a means to take account of the interaction of wastage, recruitment, promotion, and requirement. It enables the analyst to understand the impact of existing policies over the long term, and to determine changes that might be required to reach desired goals. However, the model contains 13,900 deviation variables. Although Gass (1986) has developed a process for determining priorities and weights for goal programming, there is an element of subjectivity in assigning weights or priorities to these variables to reflect the importance of meeting the corresponding goals, and there is a considerable computational burden in this procedure.

There are many other examples of goal programming models used in manpower planning. For example, Charnes and Cooper (1961) developed a model, OCMM, for the US Navy's Office of Civilian Manpower Management. Extensions of this model are described in Charnes, Cooper, and Niehaus (1971), and Charnes *et al.* (1974). Clough, Dudding, and Price (1970), and Price and Pisker (1972) have developed goal programming models for the officers of the Canadian Forces. Niehaus, Sholtz, and Thompson (1978) have introduced an interactive conversational manpower planning model based on goal programming and Markov processes. Charnes *et al.* (1978) also utilised goal programming with goals produced by a Markov transition matrix. Collins, Gass, and Rosendahl (1983) developed a model, ASCAR, to evaluate the recruitment of the US All Volunteer Armed Forces and to estimate the manpower cost for the particular policy scenario being analysed. The model has been used extensively by the Congressional Budget Office and the Office of the Assistant Secretary of Defence. Eiger *et al.* (1988) has constructed a model, MOSLS, to support manpower planning in the areas of recruitment, training and education, promotion, reclassification and re-enlistment, wastage and retirement for the US Army's enlisted force. This model has been applied by the Headquarters of the Army as an integrated manpower planning tool. A previous version of MOSLS, COMPLIP, has been described in Holz and Wroth (1980). Wijngaard (1983) has produced a model which considers the problem of aggregation. In the model, the categories of personnel are characterised by two dimensions, grade and function group. The function groups are presumed to have sufficient mobility and can be aggregated. The advantage of aggregation is that it not only reduces the forecasting load of the organisation, but also makes the forecasts more reliable. The results of the long-term aggregate model can serve as restrictions (budgets) of the short-term disaggregate model.

Goal programming is often regarded as more attractive than linear programming in manpower planning because it can cope with multiple and conflicting objectives which always exist in the real world. Nonetheless, most criticisms of goal programming models note that the planner must be able to select a set of objective function weights. Since the final solution is sensitive to the weights assigned to the objective function, successful calibration of these weights is the key element of producing acceptable solutions, but this is not easily done. In addition, as in LP manpower planning models, it may be necessary to specify acceptable ranges for promotion rate variation in each grade.

2.6.3 Interactive Augmented Weighted Tchebycheff Method

The interactive augmented weighted Tchebycheff method is a weighting vector space reduction method for solving a multiple objective programme. Detailed descriptions of this method are given in Steuer and Choo (1983) and Steuer (1986). The idea of this method is to sample a sequence of increasingly smaller subsets of the non-dominated objective function vectors until decision makers locate a solution close enough to being optimal to terminate the decision process. The sampling begins by forming a scattered group of δ-weighting vectors. Each of the δ-weighting vectors is used to construct the sample of non-dominated objective function vectors. The decision maker then selects his or her most preferred objective function vector from this generated sample. At the next iteration, another group of δ-weighting vectors is formed but centred about the weighting vector corresponding to the previous most preferred objective function vector. A new sample is formed and the most preferred objective function vector is selected again, and so forth.

In addition to using goal programming to tackle the multiple objective problems, Silverman, Steuer and Whisman (1988) have used the interactive augmented weighted Tchebycheff method to identify recruitment and promotion strategies for managing the enlisted force of the U.S. Navy. In the model, there are eleven length of service (LOS) categories, each containing three pay-grades. Individuals in their n-th year of service are in LOS class n. After completion of eleven years of service individuals are required to retire. Individuals joining the force can only enter pay-grade 1 or 2 in fixed proportions. Wastage and demotion from pay-grade i to j are calculated from a transition matrix. The purpose of the model is to determine a recruitment and promotion strategy which best meets goals pertaining to salary expenditures, promotion opportunities, mean length of service, and requirements for manpower in each of the planning periods. In the model, the promotion opportunities are concentrated on the number rather than the rate of promotions. In fact, promotion rate is better than promotion number as an indicator of promotion opportunities, since the former is a relative but the latter is an absolute number. However, using promotion rate instead of promotion number as a decision variable in the model will cause the model to be non-linear and less easy to handle.

The merit of using the interactive augmented Tchebycheff method to solve a multiple objective model is that the method avoids the burden of assigning weights as in goal programming models, and generates multiple solutions at each iteration which offer decision makers opportunities of trade-off. However, the solutions strongly depend on the δ-weighting vectors. Although decision makers can trade off these solutions, it is only a passive selection since these solutions are limited by the δ-weighting vectors. Moreover, generating these vectors is time consuming.

2.6.4 Dynamic Programming Models

Dynamic programming is an approach for solving multistage problems. Instead of dealing with decision variables simultaneously as in, for example, linear programming and goal programming, the problem is solved in stages, defined as a point at which a decision is made. In manpower planning models, this stage can be a time point, e.g. the end of year, and decision variables can be the number of recruits or the number of promotions in each year. The computations at the different stages are linked through a return function in a manner that yields a feasible optimal solution to the entire problem when the last stage is reached.

Rao (1990) developed a dynamic programming manpower planning model with the objective of minimising the manpower system costs. The model takes the manpower requirements for future periods as input and generates optimal number of recruits in terms of the cost for future periods. The relevant costs in the model consist of the following:

1. Recruitment Costs: These costs result from the process of

recruitment, and can be classified into fixed and variable costs. The fixed recruitment costs are analogous to the set-up (or ordering) costs in a production/inventory problem. The variable costs are proportional to the number of people recruited.

2. Overstaffing Costs: These are the costs associated with an under-utilised workforce. The overstaffing costs are similar to the costs of carrying inventory in an inventory system.
3. Understaffing Costs: These are the costs incurred from the non-availability of the workforce, causing decreased productivity and loss of goodwill.
4. Firing/retirement Costs: These costs result from the redundancy or retirement.
5. Retention Costs: Retention costs are the costs incurred in retaining employees in an organisation. They consist of training and development costs, probation costs, and internal mobility costs. The probation costs are those incurred owing to the learning effect of employees during a probational period.

A major difficulty with the model concerns the estimation of overstaffing and understaffing costs. In addition, the model only considers recruitment policy, while other aspects of manpower policy, such as promotion and retirement policy, are neglected.

Grinold and Stanford (1974) have also employed dynamic programming to investigate the relationship between the cost of operations and the rate of growth of an organisation; the impact of wage increases on the cost of future operations; and how the system reacts to a change in promotion policy. The objective function involves minimising the cost of operations including manpower stock and recruitment costs, and the constraints are concerned with manpower stocks and manpower budget. In the model, the promotion matrix is given and can be changed in order to investigate the effect on the manpower system. Grinold (1976) used a dynamic programming approach to deal with uncertain requirements for the US naval aviation system. In this model, manpower demand is determined by the state, defined in terms of the conflict stages with enemy, of a finite Markov chain. Individuals in the system are classified according to their length of service. The manpower supply is therefore the sum of individuals in each length of service. The objective function is to minimise the square of the error between manpower supply and demand. The constraints relate to the manpower stocks in each length of service category. Mehlmann (1980) has also developed optimal recruitment and transition strategies for manpower systems using dynamic programming.

Dynamic programming is useful in solving problems involving multiple periods or problems which can be decomposed into smaller and simpler sub-problems. However, unlike the situation with linear programming, efficient general purpose dynamic programming software is not available. Another weakness is that if the number of state variables is increased, e.g. because of multiple resources, the computational burden of solving the dynamic programming model is increased (e.g. Hartley, 1976).

2.7 SIMULATION MODELS

When small, complex or ill-defined manpower systems are concerned, aggregates or average descriptions are not sufficiently accurate to represent the systems meaningfully. In that case, a simulation approach will be appropriate. In simulation models, the characteristics or attributes of individuals, rather than groups, can be considered. These attributes can be length of service, age, qualification, gender etc. The movement of individuals can be simulated by means of a probability estimated from each person's attributes. For a new recruit entering the manpower system these attributes must be assigned from the distribution of these attributes by generating random numbers. Subsequently, the behaviour of the manpower system is projected forward.

Wishart (1976b) developed a Monte Carlo simulation model, MANSIM, which can represent different kinds of flow and complex manpower situations. Although the methods for determining promotions, the procedures for modelling mixed flows, and the use of fluid grading are the same as the KENT model (Wishart 1976a), the movements in the MANSIM are simulated in terms of individuals rather than aggregates as in the KENT model. An individual moving along a specific path is determined by his or her qualifying attributes, e.g. age, length of service, gender, and education. When a new recruit enters a manpower system, these attributes must be assigned to the recruit by generating random numbers.

Since MANSIM is used for simulating the behaviour of individuals, the approach is more flexible than other kinds of models. Furthermore, cohort studies and individual studies of, for example, career streams are possible. However, it requires far more data than other models, and the computing cost is also considerable.

There are many other examples of simulation models in manpower planning. For example, Groover (1970) developed a model, PERSYM, to assess the impact of recruitment, promotion, and assignment policies upon US enlisted personnel systems for the Office of the Secretary of Defence of the United States. Walmsley (1971) constructed a simulation model in which individuals pass through the processes of recruitment, promotion and wastage. In this model, the promotion of individuals is decided by apparent potential, which is a function of an individual's current age and salary.

Simulation models are much more flexible than any other models, since any kind of rules for deciding promotion can be incorporated, and the structure of the manpower system can be as complex as desired. However, it requires an extensive data base to operate these models and sensitivity analysis will require significant computing time.

2.8 SYSTEM DYNAMICS MODELS

In system dynamics models, the behaviour of the manpower planning system is analysed by using assumed cause and effect relationships between variables, e.g. promotion, wastage, retention, retirement, and recruitment rates, these relationships being represented by a causal-loop diagram (e.g. Clark and Lawson, 1984). The construction of this diagram is the essential first step with these models. In this step, problems will be defined and variables which influence the behaviour of the system will be identified. Once the causal-loop diagram has been developed, a computer simulation language such as DYNAMO (Pugh, 1983) can be used to analyse the behaviour of the manpower system. By using a causal-loop diagram, the relationships between factors and the essential characteristics of a system can be demonstrated clearly. Clark and Lawson claim that in comparison with entity simulation models the system dynamics approach is an economical modelling technique that needs less coding and a smaller data base.

Clark and Lawson (1984) employed a system dynamics approach to deal with the rotation and assignment problems for the US Air Force. In the model, there are three major causal-loop sectors: (1) a personnel fill sector, (2) a rotation sector, and (3) a manpower authorisation sector. The personnel fill sector deals primarily with keeping a suitable number of staff in each skill level or grade. Shortages in higher skill levels are filled from lower skill levels. The rotation sector maintains the proper number of people in overseas and stateside tours by minimising the dissatisfaction of staff. The dissatisfaction of staff arises from rotation and overseas duty time and will influence wastage rates. The manpower authorisation sector deals with the proper number and distribution of manning positions to achieve given requirements and may be considered as constants in the model. However, there are clearly difficulties in determining the dissatisfaction of staff.

Andersen and Emmerichs (1982) developed a system dynamics model for the US Department of Defense to evaluate the interaction of retirement policies with retention, compensation, and recruitment policies. The most important component in this model is voluntary wastage, which is dependent on career expectations, military pay policies, and retirement pensions. Each of these factor is represented by a multiplier, a multiplier greater than 1 indicating, for example, that voluntary wastage is greater than normal.

The main advantage of using a system dynamics approach is that the approach can highlight problems which are commonly not well understood. This advantage is obtained by involving decision makers when constructing a causal-loop diagram. However, when feedback structures are too complex, they would be like spaghetti, and neither decision makers nor analysts would be able to understand the results completely. A trade-off clearly exists between the level of aggregation and the complexity of the feedback structures, and some detailed aspects may be neglected.

2.9 OTHER APPROACHES

A number of other approaches have been used in manpower planning. For example, Perry (1978) considered the factors that influence the wastage of the secondary labour force, i.e. the urban poor and near poor. Leeson (1979) emphasised the testing and comparison of wastage by using data for the Danish State Police Force and a combined group of English Forces. Lee and Piper (1988) reported a study of the promotion processes which existed within a single region of the UK Midland Bank. Stanford (1985) focused on the movement of manpower from one grade to another, and on the factors that affect the mobility of the individuals in this process. Stanford (1980) investigated the relationships between certain characteristics of a manpower system by considering seniority in promotion and the growth (or contraction) in an organisation. McClean and Abodunde (1978) introduced a discrete-time entropy measure as an index of the stability of the length of service structure of an organisation. McClean (1986) developed a continuous-time entropy measure for labour stability. Tyler (1986) outlined the effect of organisation size on staff tenure.

2.10 DISCUSSION OF MANPOWER PLANNING MODELS

Each manpower model has its individual scope of utilisation. Since no two organisations are identical, the problems confronting the manpower planner in a specific organisation at a specific time are, as noted by Edwards (1983), unique. As a result, it seems unlikely that a universal model which can solve all kinds of manpower planning problems can be constructed. The type of model which is adopted must depend on the particular situation. It is only necessary to ensure that the model represents the real manpower system and can be used to evaluate appropriate manpower policies.

In past research on the development of manpower planning models, little attention has been given to the career development opportunities of individuals. Promotion prospects can be used as an indicator of career development opportunities. In particular, staff morale is likely to be affected if promotion rates vary significantly from one year to another. It is therefore desirable that promotion rates are as stable as possible over time. Information on the promotion opportunities will help management to review manpower policies and help individuals to make the decision on whether to remain in the organisation.

Forecasting and control are two essential aspects of manpower planning. The forecast indicates what will happen to the system if current policies continue or proposed policies are adopted. Control of a manpower system is concerned with identifying and implementing policies to achieve some desired set of goals. Bartholomew (1974) suggested that "a forecast is useful in that it may alert us to the need for action but only a theory of control can tell us how to correct the situation".

Bartholomew also noted that the emphasis had shifted from forecast to control in recent manpower planning work. Smith and Bartholomew (1988) have pointed out that there has been a need for a shift from forecasting to the investigation of more sophisticated control strategies.

Cost is generally regarded as an appropriate criterion for evaluating manpower policies. Grinold and Stanford (1974) note that if an organisation aspires to change policies and reduce costs, then knowledge of the minimum cost policy provides a direction for policy change. Indeed, changes in manpower policies can have significant effects on costs. The minimisation of manpower related costs subject to constraints on, for example, manpower requirements is therefore a suitable objective for manpower planning. In cases where this objective is adopted, optimisation models are appropriate for manpower policy evaluation.

In the manpower planning optimisation models which have been developed, promotion rate, i.e. the proportion of staff promoted per year, has not been treated as a decision variable. In previous optimisation models, the numbers promoted from each grade are used as decision variables, and these models may also consider constraints to ensure that promotion rate variation is within a specified range. However, this range of promotion rate variation must be specified in advance. In practice, the promotion rate from each grade should be treated as a decision variable, but in order to maintain staff morale, it is crucial to avoid dramatic changes in promotion rates from year to year. This research will focus on the development of multi-period manpower planning optimisation models in which promotion rates are regarded as decision variables.

2.11 SUMMARY

In this chapter, models used in manpower planning have been reviewed. These models have been classified as Markov models, renewal models, renewal models containing Markov flows, career stream models, optimisation models, simulation models, system dynamic models, and others. It seems unlikely that a universal model, which can solve all kinds of problems in manpower planning, can be constructed. The modelling framework used must depend on the particular situation. In this thesis, an optimisation approach will be adopted because of the interest in determining minimum cost recruitment, retirement and promotion policies that satisfy manpower requirements while ensuring that promotion rates remain stable over time. In this approach the promotion rate will be considered as a variable. The number of promotions must also be considered in evaluating manpower policy. The number of promotions is the product of the promotion rate and the number in a grade, both of which are variables, and therefore this manpower planning problem is non-linear. A mixed integer programming based model will be developed for this non-linear optimisation problem in Chapter 3.

CHAPTER 3

A MIXED INTEGER PROGRAMMING MODEL OF A MANPOWER SYSTEM

3.1 INTRODUCTION

In this chapter a new modelling framework is developed for manpower planning in a hierarchical organisation. The model is concerned with determining the minimum cost recruitment and promotion strategies that will satisfy the manpower needs while ensuring promotion opportunities remain stable over time. The costs considered in this approach consist of manpower stock costs, recruitment costs and service termination costs, i.e. payments to those who leave the system. In this model promotion rates, defined as the proportion of staff promoted annually, are treated as variables rather than coefficients as in other manpower planning optimisation models, and it is required that promotion rates are as stable as possible over time. This modelling approach results in a non-linear model since the number of promotions is the product of promotion rate and number in a grade, both of which are variables. A Mixed Integer Programming (MIP) based model, in which binary, i.e. variables restricted to values zero and one, are used to represent promotion decisions, is developed for this non-linear optimisation problem, and this model is solved using an iterative procedure.

3.2 MODEL DESCRIPTION AND NOTATION

The model is developed for an organisation in which recruitment may occur in all grades, internal transitions involve promotion to the next higher grade and wastage occurs in all grades. The highest grade, grade I, loses people only by wastage and the lowest grade, grade 1, obtains people only by recruitment. Demotions from any grade are not allowed. Recruitment and promotion are made to ensure that manpower levels are within acceptable ranges, i.e. the manpower requirements in each grade have upper bounds and lower bounds. It is also required that promotion rates remain stable throughout the planning period. The term wastage will be used to refer to the total loss of individuals from the organisation for whatever reason. In practice, it can be separated into voluntary and involuntary wastage. Voluntary wastage results from those who leave of their own choice. Involuntary wastage occurs whenever individuals leave for reasons beyond their own control, such as retirement, redundancy, death, and ill-health.

3.2.1 Notation

In the model, year t is defined from time t-1 to time t, where t is an integer. The number of staff in grade i, i=1,2,....,I, at the end of year t is a stock and will be denoted by n_{it}. Note that n_{it}, the number of staff in grade i at the end of year t, can also be considered as the number of staff in grade i at the start of year t+1. The numbers that move from grade i to i+1, i<I, and enter or leave the system in year t, i.e. from time t-1 to t, are flows. These flows consist of promotion, recruitment and wastage. The rates of promotion and wastage in grade i in year t are defined as proportions of the number of staff in grade i

at start of year t, i.e. $n_{i,t-1}$. Note also that the subscript t in a stock relates to the end of year t, but in a flow, it relates to an interval of year t, i.e. from t-1 to t. The diagram of stocks and flows for a manpower system is presented in diagram 2.

3.2.2 Variables

The variables in the MIP model are defined as follows:

n_{it} number of staff in grade i, $i=1,2,....,I$, at end of year t, $t=1,2,...,T$

s_t the total number of staff in the manpower system at end of year t, $t=1,2,...,T$

r_{it} number of recruits to grade i, $i=1,2,....,I$, in year t, $t=1,2,....,T$

p_{it} promotion rate from grade i, $i=1,2,....,I-1$, in year t, $t=1,2,....,T$, i.e. proportion of staff in grade i at start of year t promoted to grade i+1 in year t

3.2.3 Model Parameters

The coefficients in the MIP model are defined as follows:

N_{i0} number of staff in grade i, $i=1,2,....,I$, initially, i.e. at start of year 1

W_{it} wastage rate in grade i, $i=1,2,....,I$, in year t, $t=1,2,....,T$, as a proportion of the number of staff in post at the start of the year; the wastage rate includes loss of individuals from system for whatever reason, e.g. retirement

S_t target total number of staff in the manpower system at end of year t, $t=1,2,....,T$

E_{Ut} maximum upper proportional deviation in target total number of staff at end of year t, t=1,2,....,T

E_{Lt} maximum lower proportional deviation in target total number of staff at end of year t, t=1,2,....,T

F_{Uit} maximum upper proportional deviation in target number of staff in grade i,i=1,2,...,I, at end of year t, t=1,2,...,T

F_{Lit} maximum lower proportional deviation of target number of staff in grade i, i=1,2,...,I, at end of year t, t=1,2,...,T

G_{it} target proportion of staff in grade i, i=1,2,...,I, at end of year t, t=1,2,...,T

R_{Uit} upper bound on number of recruits to grade i, i=1,2,...,I, in year t, t=1,2,...,T

R_{Lit} lower bound on number of recruits to grade i, i=1,2,...,I, in year t, t=1,2,...,T

B_{Ui} upper bound on promotion rate from grade i to grade i+1, i=1,2,...,I-1, in any year

B_{Li} lower bound on promotion rate from grade i to grade i+1, i=1,2,...,I-1, in any year

C_{Sit} average annual salary per person in grade i, i=1,2,...,I, in year t, t=1,2,...,T

C_{Rit} average recruitment cost per person recruited to grade i, i=1,2,...,I, in year t, t=1,2,...,T

C_{Git} average service termination cost per person in grade i, i=1,2,...,I, leaving the system in year t, t=1,2,...,T

α annual discount rate

3.2.4 Objective Function

The objective function involves the minimisation of total manpower costs, i.e. the sum of manpower stock costs, recruitment costs and service termination costs.

Minimise

$$C = \sum_{i=1}^{I} \sum_{t=1}^{T} [C_{Sit}(n_{i,t-1}+n_{it})/2 + C_{Rit}r_{it} + C_{Git}W_{it}n_{i,t-1}](1+\alpha)^{-t} \qquad (3\text{-}1)$$

The manpower stock costs are the product of average annual salary per person and average number of staff. The average number of staff in a grade in a year is the number of staff at the start of the year (or at the end of previous year), i.e. $n_{i,t-1}$, plus the number of staff at the end of the year, i.e. n_{it}, divided by two. The recruitment costs include the costs incurred in the process of recruitment, for instance, cost of advertising, administrative costs and the cost of training recruits before they become fully operational. The service termination costs involve lump sum payments to those who leave the system. All costs discussed above should be discounted to their present value. For simplicity, average costs are used in the model. However, a more sophisticated cost function could be incorporated into the model.

3.2.5 Constraints

3.2.5.1 Manpower Stocks

The number of staff in grade i at end of year t, i.e. n_{it}, is equal to the manpower stock at start of the year, i.e. manpower stock at end of year t-1, less wastage and promotion from grade i, plus promotion and recruitment to grade i in year t.

$$n_{1t} = (1 - W_{1t}-p_{1t})n_{1,t-1} + r_{1t} \qquad t=1,2,\ldots,T \qquad (3\text{-}2a)$$

$$n_{It} = (1 - W_{It})n_{I,t-1} + p_{I-1,t}n_{I-1,t-1} + r_{It} \qquad t=1,2,\ldots,T \qquad (3\text{-}2b)$$

$$n_{it} = (1 - W_{it}-p_{it})n_{i,t-1} + p_{i-1,t}n_{i-1,t-1} + r_{it} \qquad (3\text{-}2c)$$

$$i=2,3,\ldots,I-1, \quad t=1,2,\ldots,T$$

$$s_t = \sum_{i=1}^{I} n_{it} \qquad t=1,2,\ldots,T \qquad (3\text{-}3)$$

Note that since p_{it} and n_{it} are variables, constraints (3-2) are non-linear. This aspect of the model will be considered later.

3.2.5.2 Total Number of Staff in the System

For the sake of acceptability of the solution, the total number of staff, s_t, in year t must be within a range defined in terms of deviations from the target total number of staff, S_t, for year t:

$$S_t(1-E_{Lt}) \le s_t \le S_t(1+E_{Ut}) \qquad t=1,2,\ldots,T \qquad (3\text{-}4)$$

where E_{Lt} and E_{Ut} are the lower and upper proportional deviations in the target total number of staff in year t respectively. The range of proportional deviation in the target total number of staff can be altered to investigate the effect on the manpower system.

3.2.5.3 Number of Staff in Each Grade

The target number of staff in grade i at the end of year t is the product of the target proportion of staff in grade i at the end of year t, i.e. G_{it}, and the target total number of staff in the manpower system at end of year t, i.e. S_t. The number of staff in each grade must be within a range defined in terms of deviations from these targets:

$$G_{it}S_t(1-F_{Lit}) \le n_{1t} \le G_{it}S_t(1+F_{Uit}) \qquad i=1,2,\ldots,I, \quad t=1,2,\ldots,T \qquad (3\text{-}5)$$

where F_{Lit} and F_{Uit} are the lower and upper proportional deviations respectively in the target number of staff in grade i in year t. The parameter G_{it} in constraint (3-5) can be changed in order to observe the influence of different grade structures on the system.

3.2.5.4 Number of Recruits

$$R_{Lit} \le r_{it} \le R_{Uit} \qquad i=1,2,\ldots,I,\ t=1,2,\ldots,T \qquad (3\text{-}6)$$

The lower and upper bounds on recruitment are assigned by the decision makers. A fluctuating recruitment in each year will cause an irregular age structure. With fluctuating recruitment, the promotion opportunities of individuals will vary because of the changing number of vacancies caused by irregular retirement, and if length of service is viewed as an indication of qualification, the number of qualified people will vary from year to year. These difficulties can be overcome by specifying a narrow range for recruitment, i.e. a stable recruitment policy.

3.2.5.5 Stable Promotion Rates

$$B_{Li} \le p_{it} \le B_{Ui} \qquad i=1,2,\ldots,I-1,\ t=1,2,\ldots,T \qquad (3\text{-}7)$$

Promotion rates are variables but it is desirable for them to be as stable as possible over time since staff morale is likely to be affected if promotion opportunities vary significantly from one year to another. For example, if it is found necessary substantially to reduce promotion rates so that staff cannot be promoted even though they are qualified, then some staff may become frustrated and leave, i.e. wastage rates may also increase as individuals feel their prospects of promotion are reduced. If the better staff leave, the overall quality of the organisation, in terms of the experience and training of the staff, will suffer. This may subsequently create good promotion opportunities for those who enter the organisation later and for those who may not yet have the required qualifications. These unequal promotion opportunities are likely to be perceived as being unfair. For these reasons, it is wise to minimise the fluctuations in promotion rates from year to year. This dynamic stability can be achieved by setting a narrow range for the fluctuations in annual promotion rates. The upper bound and lower bound of this range in each grade, i.e. B_{Ui} and B_{Li} respectively, can be determined by use of the model rather than set subjectively by decision makers. A method for using the model to determine appropriate values for B_{Ui} and B_{Li} will be discussed later.

3.3 USE OF BINARY VARIABLES IN THE NON-LINEAR OPTIMISATION PROBLEM

The non-linear aspect of the model noted in constraints (3-2) causes considerable solution difficulties. These difficulties, however, can be dealt with by using binary variables, i.e. variables confined to values 0 and 1, and by defining a set of ranges for the promotion rate, p_{it}, from grade i in year t, with the same set of ranges of promotion rate from grade i being used in each year. The requirement for stable promotion rates over time can then be modelled by imposing constraints for each grade to ensure that the promotion rate from that grade is in the same range in each year. A similar approach has been used by Glen (1991) in modelling fishing activities. By using this approach the promotion opportunities in each year will remain as stable as required, and the minimum cost recruitment and promotion strategies can be determined.

Assume that J ranges are used to define the ranges for the promotion rate, p_{it}, from grade i to grade i+1 in any year t, such that

range 1: B_{1i} to B_{2i}

range 2: B_{2i} to B_{3i}

.

.

.

range J: B_{Ji} to $B_{J+1,i}$

where $B_{1i}, B_{2i},\ldots,B_{J+1,i}$ are the bounds of the promotion ranges for grade i, i=1,2,...,I-1, in each year, t, t=1,2,...,T, with $B_{ji} < B_{j+1,i}$, j=1,2,...,J, $B_{1i} \geq 0$ and $B_{J+1,i} \leq 1$. These J ranges will be referred to as the possible ranges.

Let m_{it} denote the number of staff promoted from grade i to grade i+1, i=1,2,...,I-1, in year t, t=1,2,...,T, then

$$m_{it} = p_{it}n_{i,t-1} \qquad i=1,2,\ldots,I-1,\ t=1,2,\ldots T.$$

The manpower stock constraints (3.2) then become:

$$n_{1t} = (1 - W_{1t})n_{1,t-1} - m_{1t} + r_{1t} \qquad \forall t \qquad (3\text{-}8a)$$

$$n_{It} = (1 - W_{It})n_{I,t-1} + m_{I-1,t} + r_{1t} \qquad \forall t \qquad (3\text{-}8b)$$

$$n_{it} = (1 - W_{it})n_{i,t-1} - m_{it} + m_{i-1,t} + r_{it} \quad i=1,2,\ldots,I-1,\ \forall t \qquad (3\text{-}8c)$$

The requirement for stability in promotion rates can be modelled by defining binary variables δ_{ji} such that

$$\delta_{ji} = 1 \Rightarrow B_{ji}n_{i,t-1} \leq m_{it} \leq B_{j+1,i}n_{i,t-1}$$

with $$\sum_{j=1}^{J} \delta_{ji} = 1$$

where i=1,2,...,I-1, j=1,2,...,J, t=1,2,...,T.

Constraints for the definition of δ_{ji} can be derived in a number of ways. For example, we require that

when $\delta_{ji}=1$ then $m_{it}-B_{ji}n_{i,t-1} \geq 0$ and $m_{it}-B_{j+1,i}n_{i,t-1} \leq 0$

and when $\delta_{ji}=0$ then $m_{it}-B_{ji}n_{i,t-1} \geq -M$ and $m_{it}-B_{j+1,i}n_{i,t-1} \leq M$

where M is a sufficiently large number. Thus the constraints for stable promotion rates (3-7) become:

$$m_{it} - B_{ji}n_{i,t-1} \geq -M(1-\delta_{ji})$$

and $$m_{it} - B_{j+1,i}n_{i,t-1} \leq M(1-\delta_{ji})$$

i.e. $$-m_{it} + B_{ji}n_{i,t-1} + M\delta_{ji} \leq M \qquad \forall\ j,t,\ i=1,2,\ldots,I-1 \qquad (3\text{-}9)$$

and $$m_{it} - B_{j+1,i}n_{i,t-1} + M\delta_{ji} \leq M \qquad \forall\ j,t,\ i=1,2,\ldots,I-1 \qquad (3\text{-}10)$$

with $$\sum_{j=1}^{J} \delta_{ji} = 1 \qquad (3\text{-}11)$$

where $0 \leq \delta_{ji} \leq 1$ and δ_{ji} are integers, i.e. δ_{ji} are binary variables.

By using constraints of form (3-9), (3-10) and (3-11) an MIP model of a manpower system can be developed. The choice of M has been discussed by, for example, Camm, Raturi, and Tsubakitani (1990). In this case, the value of M can be determined by noting that if $\delta_{ki}=1$, $1 \leq k \leq J$, $i=1,2,\ldots,I-1$, then for $j=k$,

$$m_{it} \geq B_{ki}n_{i,t-1} \qquad (3\text{-}12)$$

and $$m_{it} \leq B_{k+1,i}n_{i,t-1} \qquad (3\text{-}13)$$

and for $j \neq k$, since $\delta_{ji}=0$,

$$m_{it} \geq B_{ji}n_{i,t-1} - M \qquad j \neq k \qquad (3\text{-}14)$$

and $$m_{it} \leq B_{j+1,i}n_{i,t-1} + M \qquad j \neq k \qquad (3\text{-}15)$$

In order to ensure that constraints (3-14) and (3-15) are redundant, it is required that

$$B_{ji}n_{i,t-1} - M \leq B_{ki}n_{i,t-1}, \quad \text{i.e. } M \geq (B_{ji} - B_{ki})n_{i,t-1}$$

and $$B_{j+1,i}n_{i,t-1} + M \geq B_{k+1,i}n_{i,t-1}, \quad \text{i.e. } M \geq (B_{k+1,i} - B_{j+1,i})n_{i,t-1}$$

Since $B_{ji} \leq 1$, then by setting

$$M = \max_{\substack{i=1,2,\ldots,I-1,\\ t=1,2,\ldots,T}} [\, n_{i,t-1} \,] \qquad (3\text{-}16)$$

it is ensured that (3-14) and (3-15) are redundant. More precisely,

$$M = \left(\max_{i=1,2,\ldots,I-1} [B_{Ji}-B_{1i}] \right) \left(\max_{\substack{i=1,2,\ldots,I-1\\ t=1,2,\ldots,T}} [n_{i,t-1}] \right) \qquad (3\text{-}17)$$

where B_{1i} is the lower bound on the first possible range for the promotion rate from grade i to grade i+1; B_{Ji} is the lower bound for the Jth possible range of the promotion rate from grade i to grade i+1.

Clearly, by using binary variables to equalise promotion opportunities in this way it is only possible to get an approximate solution. The accuracy of this approximation can be improved by using narrower range intervals, and therefore increasing the number of possible ranges for p_{it}. However, obtaining an accurate approximation for equalising the promotion rate, p_{it}, over T years will require a large number of binary variables, since a binary variable, δ_{ji}, is required for each range j, j=1,2,...J, in each grade i, i=1,2,...,I-1. In general, the computational time to solve an MIP model depends on the number of variables and the number of constraints. Therefore the solution of an MIP model which incorporates a large number of binary variables will involve a substantial computational load. Moreover, with narrow range intervals for p_{it} there may be no feasible solution, since constraints, (3-9) and (3-10), limiting the number of staff promoted in each year may conflict with other constraints such as the target number of staff in each grade, i.e. constraint (3-5). Note, however, that in practice it is only necessary to find solutions in which the promotion rates from grade i are stable to within some specified range of variation, e.g. ± 5%, which is acceptable to the management of the system.

Because of the difficulties in solving a single MIP model which contains a large number of binary variables for equalising promotion rates, an iterative solution procedure is used. In this procedure, a limited number of possible ranges for promotion rates is used and the range width is reduced at successive iterations until either the range of promotion rates is acceptable to the decision makers or an infeasible solution is found.

3.4 THE MIP MODEL

The entire MIP model for iteration k, k=1,2,..., using bounds $B_{ji}^{(k)}$ for promotion rate range for grade i, i=1,2,...,I-1, j=1,2,...,J, is summarised below. The definitions of the variables and parameters of this model are presented in appendix A.

Minimise

$$\sum_{i=1}^{I} \sum_{t=1}^{T} [C_{Sit}(n_{i,t-1}+n_{it})/2 + C_{Rit}r_{it} + C_{Git}W_{it}n_{i,t-1}](1+\alpha)^{-t} \qquad (3\text{-}18a)$$

subject to

$$n_{1t} - (1 - W_{1t})n_{1,t-1} + m_{1t} - r_{1t} = 0 \qquad \forall\ t \qquad (3\text{-}18b)$$

$$n_{It} - (1 - W_{It})n_{I,t-1} - m_{I-1,t} - r_{It} = 0 \qquad \forall\ t \qquad (3\text{-}18c)$$

$$n_{it} - (1 - W_{it})n_{i,t-1} + m_{it} - m_{i-1,t} - r_{it} = 0 \qquad \forall\ t,\ 2\leq i\leq I-1 \qquad (3\text{-}18d)$$

$$s_t - \sum_{i=1}^{I} n_{it} = 0 \qquad \forall\ t \qquad (3\text{-}18e)$$

$$s_t \geq S_t(1-E_{Lt}) \qquad \forall\ t \qquad (3\text{-}18f)$$

$$s_t \leq S_t(1+E_{Ut}) \qquad \forall\ t \qquad (3\text{-}18g)$$

$$n_{it} \geq G_{it}S_t(1-F_{Lit}) \qquad \forall\ i,t \qquad (3\text{-}18h)$$

$$n_{it} \leq G_{it}S_t(1+F_{Uit}) \qquad \forall\ i,t \qquad (3\text{-}18i)$$

$$r_{it} \geq R_{Lit} \qquad \forall\ i,t \qquad (3\text{-}18j)$$

$$r_{it} \leq R_{Uit} \qquad \forall\ i,t \qquad (3\text{-}18k)$$

$$-m_{it} + B_{ji}^{(k)}n_{i,t-1} + M\delta_{ji} \leq M \qquad \forall\ j,t,k,\ i=1,2,\ldots,I-1 \qquad (3\text{-}18l)$$

$$m_{it} - B_{j+1,i}^{(k)}n_{i,t-1} + M\delta_{ji} \leq M \qquad \forall\ j,t,k,\ i=1,2,\ldots,I-1 \qquad (3\text{-}18m)$$

$$\sum_{j=1}^{J} \delta_{ji} = 1 \qquad i=1,2,\ldots,I-1 \qquad (3\text{-}18n)$$

$$n_{it},\ s_t,\ r_{it},\ m_{it} \geq 0, \quad \delta_{ji}=0,1$$

3.5 THE ITERATIVE SOLUTION PROCEDURE

The iterative procedure for solving the MIP model (3-18) of a hierarchical manpower system is described below:

Step 1:

Set iteration number k to 1, define a set of J possible ranges for promotion rate, p_{it}, in grade i, i=1,2,...,I-1, in any year t, i.e. assign lower bound and upper bound values for range j, i.e. $B_{ji}^{(1)}$ and $B_{j+1,i}^{(1)}$ in constraints (3-18l) and (3-18m). These bound values can be decided simply by dividing the promotion rate range, 0 to 1, into equal widths or using experience. Go to step 2.

Step 2:

Solve the MIP model for iteration k, to determine which of the J possible ranges in each grade minimises the objective function. This range for the promotion rate, p_{it}, in grade i in any year t will be referred to as the optimal range for grade i at iteration k. Define the upper bound and lower bound of the optimal range for grade i at iteration k as $U_i^{(k)}$ and $L_i^{(k)}$ respectively. The range width of the optimal range for each grade at iteration k, $H^{(K)}$, is given by

$$H^{(k)} = U_i^{(k)} - L_i^{(k)}$$

If the optimal range is acceptable to the decision makers, or an infeasible solution is found, then stop; otherwise go to step 3. This optimal range of width which is acceptable to the decision makers will be referred to as the acceptable range.

Step 3:

The range width, $H^{(k+1)}$, for the next iteration, i.e. iteration k+1, is then

$$H^{(k+1)} = QH^{(k)}$$

where Q, 0<Q<1, is the reduction factor for reducing the range width between successive iterations. At iteration k+1 the overall range interval for promotion rate, p_{it}, in grade i in any year t is equal to the number of possible ranges, J, multiplied by the range width, $H^{(k+1)}$. To reduce the possibility of the procedure finding a local optimal, the overall range for promotion rate, p_{it}, at iteration k+1 should overlap ranges adjacent to the optimal range at iteration k. The total overlap, i.e. $JH^{(k+1)}-H^{(k)}$, should, where possible, be equally distributed to the left and right of the optimal range at iteration k. At iteration k+1, the end overlap to the left or right of the optimal range at iteration k will be defined as $\Omega^{(k+1)}$, and is equal to half the total overlap.

The overall range at iteration k+1 for grade i is divided into J ranges of equal width, where the range width of each range in grade i at iteration k+1 is $H^{(k+1)}$. The lower and upper bounds of the ranges in each grade at iteration k+1, i.e. $B_{ji}^{(k+1)}$ and $B_{j+1,i}^{(k+1)}$, are substituted for $B_{ji}^{(k)}$ and $B_{j+1,i}^{(k)}$ in constraints (3-18l) and (3-18m) of the MIP model. The method for calculating these bounds is described in section 3.6. Then go to step 2.

3.6 CHOICE OF RANGE BOUND VALUES

The bound values of the J ranges of equal width for promotion rate, p_{it}, in grade i in any year t at iteration k+1 are affected by the number of ranges, J, the upper and lower bounds of the optimal range at iteration k, i.e. $U_i^{(k)}$ and $L_i^{(k)}$ respectively, and the reduction factor, Q, used to reduce the range width between successive iterations. Where possible, the total overlap, i.e. $JH^{(k+1)}-H^{(k)}$, is equally distributed to the left and right of the optimal range at iteration k, in order to maintain the median of the optimal range of iteration k in the middle of the overall range at iteration k+1.

The symbols used in the procedure for determining the range bound values at each iteration are:

J — the number of possible ranges for promotion rate, p_{it}, in grade i, i=1,2,...,I-1, in any year

$L_i^{(k)}$ — lower bound of the optimal range for promotion rate, p_{it}, in grade i, i=1,2,...,I-1, in any year at iteration k, k=1,2,...

$U_i^{(k)}$ — upper bound of the optimal range for promotion rate, p_{it}, in grade i, i=1,2,...,I-1, in any year at iteration k, k=1,2,...

$B_{ji}^{(k+1)}$ — lower bound of promotion rate range j, j=1,2,...,J, from grade i to grade i+1, i=1,2,...,I-1, in any year at iteration k+1, k=1,2,...

$B_{j+1,i}^{(k+1)}$ — upper bound of promotion rate range j, j=1,2,...,J, from grade i to grade i+1, i=1,2,...,I-1, in any year at iteration k+1, k=1,2,...

Q — reduction factor, i.e. the proportion by which the promotion rate range width in each grade is reduced between iterations k and k+1, k=1,2,...

$H^{(k)}$ — promotion rate range width in each grade in any year at iteration k

$\Omega^{(k+1)}$ — end overlap at iteration k+1, i.e. the amount by which the overall range at iteration k+1 overlaps equally to the left and right of the optimal range at iteration k, k=1,2,...

From the above definitions,

$$H^{(k+1)} = ((U_i^{(k)} + \Omega^{(k+1)}) - (L_i^{(k)} - \Omega^{(k+1)}))/J \quad i=1,2,\ldots,I-1 \qquad (3\text{-}19)$$

Since $H^{(k+1)} = QH^{(k)}$

$$\Omega^{(k+1)} = H^{(k)}(QJ-1)/2 \qquad i=1,2,\ldots,I-1 \qquad (3\text{-}20)$$

Clearly, $\Omega^{(k+1)} \geq 0$, $H^{(k)} \leq 1$, $Q < 1$ and $J > 1$. The values of Q and J must be specified for an application of the model, and from (3-20) it is clear that

$$Q \geq 1/J \qquad (3\text{-}21)$$

The bound values of the J possible ranges at iteration k+1, i.e. $B_{ji}^{(k+1)}$, for grade i, i=1,2,...,I-1, can therefore be determined as in one of the following three cases.

Case 1: $U_i^{(k)}+\Omega^{(k+1)} \geq 1$

$$B_{J+1,i}^{(k+1)} = 1$$

$$B_{Ji}^{(k+1)} = 1-H^{(k+1)}$$

$$B_{J-1,i}^{(k+1)} = 1-2H^{(k+1)}$$

$$\vdots$$

$$B_{1i}^{(k+1)} = 1-JH^{(k+1)}$$

Case 2: $U_i^{(k)}+\Omega^{(k+1)}<1$ and $L_i^{(k)}-\Omega^{(k+1)}\leq 0$

$$B_{1i}^{(k+1)} = 0$$

$$B_{2i}^{(k+1)} = H^{(k+1)}$$

$$\vdots$$

$$B_{J+1,i}^{(k+1)} = JH^{(k+1)}$$

Case 3: $U_i^{(k)}+\Omega^{(k+1)}<1$ and $L_i^{(k)}-\Omega^{(k+1)}>0$

$$B_{1i}^{(k+1)} = L_i^{(k)} - \Omega^{(k+1)}$$

$$B_{2i}^{(k+1)} = L_i^{(k)} - \Omega^{(k+1)}+H^{(k+1)}$$

.
.
.

$B_{J+1,i}^{(k+1)} = L_i^{(k)} - \Omega^{(k+1)} + JH^{(k+1)}$

Note that in the iterative solution procedure, the number of possible ranges for the promotion rate in each grade and the reduction factor, i.e. J and Q respectively, are specified and then the end overlap value, $\Omega^{(k+1)}$ is determined. Alternatively, the end overlap value at iteration k+1 can also be specified as a fixed proportion, defined as V, of the range width at iteration k, i.e. $\Omega^{(k+1)}=VH^{(k)}$. When V and J are given, the reduction factor, Q, then can be determined by

$$Q = (2V+1)/J \qquad (3\text{-}22)$$

which is derived from equation (3-20).

To demonstrate the calculation of the bound values of the J possible ranges, assume that the upper bound, $U_2^{(k)}$, and the lower bound, $L_2^{(k)}$, of the optimal range for promotion rate, p_{2t}, in grade 2 at iteration k are 0.4 and 0.6 respectively. At iteration k+1, the overall range is divided into 3 possible ranges and the width of each range is 50% of the optimal range at iteration k, i.e. J=3 and Q=0.5. Therefore, the bound values of each possible range can be calculated as follow:

$H^{(k)} = U_2^{(k)} - L_2^{(k)} = 0.2$

$H^{(k+1)} = QH^{(k)} = 0.1$

$\Omega^{(k+1)} = H^{(k)}(QJ-1)/2 = 0.05$

Since $U_2^{(k)}+\Omega^{(k+1)}<1$ and $L_2^{(k)}-\Omega^{(k+1)}>0$, case 3 is used to calculate these bound values, that is

range 1	0.35 to 0.45
range 2	0.45 to 0.55
range 3	0.55 to 0.65

Note that the overall range at iteration k+1, from 0.35 to 0.65, is 50%

wider than the optimal range at iteration k.

3.7 A NUMERICAL ILLUSTRATION

The use of the model is now demonstrated using illustrative data for the officer ranks in a military system.

3.7.1 Problem Description

The officers are divided into six grades or ranks: second lieutenant, subaltern, captain, major, lieutenant-colonel, and colonel or higher grades. The ranks above colonel are lumped together with colonel because of the small numbers involved. These six grades will be referred to as grade 1 to grade 6 respectively for convenience. Recruitment occurs only into grade 1 and the recruits must be trained for four years before they become fully operational. In other words, the recruitment in year t in the model represents recruitment which occurred actually in year t-4, i.e. four years earlier. Promotion from grade i is made only into the grade immediately above, i.e. i+1, i=1,2,...5. Wastage is assumed to depend on grade and includes retirement and death. The total manpower level and the manpower level in each grade are allowed to vary within specified ranges. It is also required that the promotion rates remain stable throughout the planning period. The problem is concerned with determining minimum cost recruitment and promotion strategies that satisfy the manpower requirements while ensuring promotion rates remain stable over the next ten years.

3.7.2 Data for the Model Parameters

It is assumed that the target total number of staff, S_t, the maximum upper and lower proportional deviations in target total number of staff, E_{Ut} and E_{Lt}, the maximum upper and lower proportional deviations in target number of staff in grade i, F_{Uit} and F_{Lit}, the upper and lower bounds on the number of recruits to grade 1, R_{U1t} and R_{L1t}, and annual discount rate, α, keep the same values in each year t. The values of these parameters are given below:

$S_t = 10000 \qquad \alpha = 0.1$

$E_{Ut} = 0.05 \qquad E_{Lt} = 0.1$

$F_{Uit} = 0.05 \qquad F_{Lit} = 0.1$

$R_{U1t} = 900 \qquad R_{L1t} = 800$

The wastage rate, W_{it}, average salary per person, C_{Sit}, and average recruitment cost per person, C_{R1t}, are given in tables 3.1 to 3.3 respectively. The initial number of staff, n_{i0}, target proportion of staff in grade i, G_{it}, and average service termination cost per person as a function of average salary, are given in table 3.4.

3.7.3 Solution of the Model

To avoid an inappropriate initial choice of the possible ranges for promotion rate, a wide range width for each range in each grade is specified for the first iteration. Therefore, at the first iteration two possible ranges for the promotion rate, p_{it}, in each grade i in any year t are considered. The width of each range is 0.5, thus the bound values are:

$B_{1i}^{(1)} = 0;\ B_{2i}^{(1)} = 0.5;$

$B_{3i}^{(1)} = 1;\ i=1,2,\ldots,5.$

These bound values are assigned to constraints (3-181) and (3-18m). The model was set up and solved using XPRESS-MP (Dash associates, 1991), a PC based MIP software product, on a 386SX-16 laptop PC. XPRESS-MP contains three modules - a model builder (or matrix generator), an optimiser and a report writer. The model builder allows the model to be specified in a form similar to the mathematical statement of the model, and creates a matrix file for input to the optimiser.

At the first iteration, the promotion rates, p_{it}, in the optimal solution fall into the following optimal ranges in each year t, t=1,2,...,10:

$$0 \le p_{1t} \le 0.5$$
$$0 \le p_{2t} \le 0.5$$
$$0 \le p_{3t} \le 0.5$$
$$0 \le p_{4t} \le 0.5$$
$$0 \le p_{5t} \le 0.5$$

It is assumed that these optimal ranges are not sufficiently narrow, and therefore each of these optimal ranges is reduced by a factor of 2 and divided it into four ranges, i.e. Q=0.5 and J=4, at successive iterations until either an acceptable solution or an infeasible solution is obtained. Note that by using this strategy the overall range at iteration k+1, k≥1, will overlap the ranges adjacent to the optimal range at iteration k by 100%. A computer program in BASIC (see appendix C3) was developed to obtain the bound values at each iteration. This program automatically transfers the values to a file for input to the model builder in XPRESS-MP. Moreover, the program can change the output format of the solution to the MIP model to import the results into Harvard Graphics (Campbell, 1990) software for graphical presentation at each iteration.

The results in terms of the total cost and range width at each iteration are given in table 3.5a. The optimal ranges in each grade at iteration 2, 3 and 4 are presented in table 3.5b. Note that in table 3.5a at run 5, i.e. iteration 5, there is no feasible solution. This suggests that the width of these possible ranges at iteration 5 is too narrow, and that the solution at iteration 4 is a final solution. In Chapter 4 a method for continuing the search for a feasible solution in a narrower range, such as that used in iteration 5, is described. In order to check that the solution at iteration 4 is a global, rather than a local optimal, for the range width used in iteration 4, a further nine promotion rate ranges adjacent to the optimal range at iteration 4, and of the same width, were defined. These ten promotion rate ranges are presented in table 3.6. The MIP model for these ten ranges was then solved and it was found that the solution is the same as at iteration 4 indicating that for a given set of bounds the iterative procedure finds the global optimum. The final results are summarised in figures 3.1 to 3.11 and in table 3.7.

From figure 3.3, it can be seen that the total number of staff in each year is stable and around the target number, 10000. However, in figure 3.1 it can be seen that in most years the number of staff in lower grades, i.e. grade 1 to grade 3, are above their target numbers, 2000, 2000, and 2500 respectively, while staff numbers in higher grades, i.e. grade 4 to grade 6 are below their target numbers, 2000, 1000, and 500 respectively. This phenomenon could result from the fact that service termination costs, as noted in table 3.4, in the higher grades are much greater than in the lower grades. It could be beneficial to maintain higher numbers of staff in the lower grades than that in the higher grades in terms of cost. The promotion rates in the lower grades, as presented in figures 3.7 and 3.8, fluctuate near the upper bounds and in the upper grades the promotion rates, as presented in figure 3.11, fluctuate near the lower bounds. In table 3.7 it can be seen that the lowest annual manpower cost is £K240,291 in year 5 and the greatest annual manpower cost is £K278,340 in year 2, i.e. 15.8% greater than the lowest cost. Figure 3.6 shows that the service termination cost is approximately 38% of the manpower cost. This figure seems to be very high, suggesting that it may be desirable to consider modifications to the cost system.

3.8 SUMMARY

In this chapter a Mixed Integer Programming (MIP) approach has been used to tackle a non-linear manpower planning problem. The non-linear nature of the problem arises because the number promoted from grade i to grade i+1 is the product of the promotion rate and the number in grade i, both of which are variables. In the MIP model, promotion rates are considered as variables, and constraints are imposed to keep promotion rates as stable as far as possible over time.

The non-linear manpower planning problem can be modelled by defining binary variables associated with a set of possible ranges for the promotion rate from each grade. An iterative solution procedure, in which the range width is reduced at successive iterations, is used until either an infeasible solution is found or the range of promotion rate variation is acceptable to the decision makers.

Clearly, the number of possible ranges for promotion rate in each grade and the choice of the factor by which the range width is reduced at successive iterations, will influence the solution of the model. In order to reduce the possibility of finding a local, rather than the global, optimum, the overall range at iteration k+1 overlaps the optimal range at iteration k. The results obtained indicate that the iterative procedure will find the global optimal for a given set of promotion rate range bounds. However, it is desirable to investigate the effect of the choice of overlap. Moreover, even if a solution with an acceptable range for promotion rate in each grade is found, a better, i.e. lower cost, solution with the same range width may be obtained by changing the range bounds. Similarly, an infeasible solution caused by an inappropriate promotion rate range may become feasible by specifying another range of the same width. In addition, since the MIP model may contain many integer variables, the computational time becomes crucial. All of these factors will be discussed in Chapter 4.

CHAPTER 4

COMPUTATIONAL ASPECTS OF THE MIP MODEL

4.1 INTRODUCTION

For a given set of manpower requirements over a specified planning horizon, the MIP model of the hierarchical manpower system is used to determine the cost minimising promotion rate p_{it}, defined in terms of the proportion of staff in grade i at start of year t promoted to grade i+1 during year t, such that for each grade i the promotion rate p_{it} is approximately constant from year to year, i.e. for all t. The model is solved using an iterative procedure in which a limited number of promotion rate ranges are considered, with the width of the range of permitted promotion rates being reduced at successive iterations, until a solution which is acceptable, in terms of the range of variation in promotion rates, is obtained. When this approach is applied to a problem involving a specified number of periods, the solution and computational effort in solving the problem will depend on the number of promotion rate ranges at each iteration, the factor for reducing the width of these ranges between successive iterations, the value of M in constraints (3-18l) and (3-18m), and the branch and bound solution strategy, e.g. the priority of integer variables in branching, and the use of special ordered sets. The effect of these factors on both the solution and the computational effort is now considered. In addition, a method of finding an improved solution for a specified range width is developed.

4.2 THE NUMBER OF RANGES AND THE RANGE WIDTH REDUCTION FACTOR

Since the solution of the MIP model is influenced by the bound values of the possible ranges for the promotion rate p_{it} in each grade i at each iteration, it is important to determine suitable values for the number of possible ranges, J, and the reduction factor, Q, for reducing the width of the ranges between successive iterations. Because of the difficulty in determining suitable values for J and Q, a strategy of overlapping the possible ranges adjacent to the left and the right of the optimal range at iteration k is used. The effect on the solution of the choice of values for the number of possible ranges, J, reduction factor, Q, and the overlap will be considered by conducting some numerical experiments. The data in these experiments are the same as in the numerical illustration of section 3.7.

4.2.1 Numerical Experiments

The effect on the solution of the choice of values for the number of ranges, reduction factor and the overlap was investigated in three sets of numerical experiments. In these experiments, the ranges for even values of $J \geq 4$ (or odd values of $J \geq 5$) were obtained by adding equal width ranges adjacent to the extreme ranges for the case with J-2 ranges, so that, for example, the range bounds for J=3 are a subset of the range bounds for J=5. For instance, suppose that at the first iteration the optimal promotion rate range from grade 1 to grade 2 is 0.25 to 0.5. At the second iteration, for J=2 and Q=0.5, this optimal range is divided into two possible ranges, the width of each range being 50% of the optimal range at iteration 1, and the two ranges for promotion rate from grade 1 to grade 2 at iteration 2 are:

range 1: 0.25 to 0.375

range 2: 0.375 to 0.5

For J=4, the four ranges are obtained by adding another two equal width ranges adjacent to the right and left of the two ranges used for J=2, that is

range 1: 0.125 to 0.25

range 2: 0.25 to 0.375

range 3: 0.375 to 0.5

range 4: 0.5 to 0.625.

In the first set of numerical experiments, the impact on the solution of the choice of the number of ranges, J, and reduction factor, Q, were investigated using J=3, 4, 5 with Q=0.4 for each value of J. In this case, at iteration k+1, $k \geq 1$, the range width for the promotion rate p_{it} in each grade i at iteration k+1, $k \geq 1$, is 40% of the range width at iteration k. From equation (3-20) it can be seen that for J=3, 4 and 5, the end overlaps are 10%, 30%, and 50% respectively, i.e. the overall range at iteration k+1 will overlap the ranges to the left and the right of the optimal range at iteration k by 10%, 30%, and 50% respectively. Note that the case of J=2 was not considered since, from (3-21), Q must be greater than or equal to 1/J. At the first iteration in all experiments, four ranges for the promotion rate p_{it} in each grade i are given, namely 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75 and 0.75 to 1. In subsequent iterations the range width is reduced by a factor Q=0.4. Thus, for example, at iteration 2, the width of the ranges in each grade is 0.1, i.e. $H^{(2)}=QH^{(1)}=0.4*0.25=0.1$.

The results for the first set of experiments are presented in table 4.1a. Note that with this range width reduction procedure, the range bounds for J=3 are a subset of the range bounds for J=5. In table 4.1a at run 2, i.e iteration 2, it can be seen that the cost for 30% overlap, i.e. J=4, is higher than for 10% overlap, i.e. J=3. However, the case at run 3 contrasts with that at run 2, where the cost for J=4 is lower than the cost for J=3 and J=5. At run 4, when the range width is 0.06, no feasible solution can be found in all experiments. Note that the results for J=3 and J=5 are the same. The optimal ranges of each grade for 40% reduction in range width between iterations are presented in table 4.1b.

In the second set of numerical experiments, the impact on the solution of the choice of values for J was investigated using J=2, 3, 4 and 5 with Q=0.5 for each value of J. For J=2, 3, 4 and 5, the end overlap at iteration k+1, i.e. the amount by which the overall range at iteration k+1 overlaps equally to the left and right of the optimal range at iteration k, will be 0%, 25%, 50%, and 75% respectively. Note that with this range width reduction procedure, the range bounds for J=2 are a subset of the range bounds for J=4, and the range bounds for J=3 are a subset of the range bounds for J=5. The results for this set of experiments are presented in table 4.2a. Note that at each iteration or run, the results for an even number of ranges, i.e. J=2 and J=4 are the same, and the results for an odd number of ranges, i.e. J=3 and J=5 are the same but different from the results for cases with an even number of ranges. In table 4.2a, it can be seen that at iterations 2 and 3, the cost for an even number of ranges is lower than the cost for an odd number of ranges. In particular the cost for no overlap, i.e. J=2, is lower than for 25% overlap, i.e. J=3. At iteration 4, i.e. for range width 0.0313, no feasible solution can be found for the cases with J=2 and J=4. However, a feasible solution with the same range width can be found for the J=3 and J=5 cases. The optimal ranges in each grade for the case where the range width is reduced by 50% between iterations are presented in table 4.2b.

In the third set of numerical experiments, the model was solved using J=2, 3, 4 and 5 with Q=0.6 for each value of J. The end overlaps for J=2, 3, 4 and 5 are the overall range at iteration k+1 overlaps the ranges to the left and the right of the optimal range of iteration k by 10%, 40%, 70%, and 100% respectively, and again the range bounds for J=2 are a subset of the range bounds for J=4, and the range bounds for J=3 are a subset of the range bounds for J=5. The results for this set of experiments are presented in table 4.3a. Again it can be seen that at each iteration the results for an even number of ranges, i.e. J=2 and J=4, are the same, and the results for an odd number of ranges, i.e. J=3 and J=5 are the same but different from the results for cases with an even number of ranges. In table 4.3a, it shows that at iteration 2, the cost for J=2 and J=4 is lower than the cost for J=3 and J=5. At iteration 3, i.e. range width 0.09, no feasible solution can be found for all experiments. The optimal ranges of each grade for 60% reduction in range width between iterations are presented in table 4.3b.

4.2.2 Observations from the Numerical Experiments

The following observations are made from the results in this set of computational experiments:

1. In the range width reduction procedure, the range bounds for J=2 are a subset of the range bounds for J=4, and the range bounds for J=3 are a subset of the range bounds for J=5. From the results for a

given range width, it can be seen that before an infeasible solution is found, the results for the runs with an even numbers of ranges, i.e. even values of J, are the same, and similarly for odd values of J. This suggests that a suitable choice of value for J is either 2 or 3. Note, however, that if an infeasible solution is found when using a small value of J, it may be possible to find a feasible solution when a larger value is used.

2. There is no evidence to show that increasing the overlap value will yield a lower cost solution. This means that for a given range width, increasing the number of possible ranges, J, does not guarantee a better solution. For instance in table 4.1a, when the range width is 0.1, the cost for J=3, i.e. 10% overlap, is smaller than for J=4, i.e. 30% overlap. However, when the range width is 0.04, the cost for J=3 is larger than for J=4.

3. The results show, as was found in Chapter 3, that by adding equal width ranges adjacent to an optimal range and solving the problem again the solution does not change. This observation is also consistent with the results in table 3.6, in which nine equal width ranges adjacent to the optimal range in each grade are specified. Since adding equal width ranges adjacent to the optimal ranges does not change the solution, this indicates that the iterative solution procedure described in section 3.5 converges. Therefore, it can be inferred that for a specified range width, the optimal range of the global optimal solution is near the optimal range generated by using the iterative solution procedure.

4.3 A STRATEGY FOR SEARCHING FOR AN IMPROVED SOLUTION

It has been noted that a suitable value for the number of possible ranges, J, is either 2 or 3. The iterative procedure described in section 3.5 is used to find an optimal solution in which the promotion rate range width is acceptable to the decision makers, or until the procedure stops because no feasible solution with an acceptable range width can be found. However, even when an acceptable solution is found, it may be possible to find a lower cost solution. In cases where no acceptable feasible solution is found, it may be possible to find a feasible solution with an acceptable range width by specifying other sets of range bounds for the promotion rates.

The numerical experiments of section 4.2 show that the solution is affected by the bounds of the possible ranges since, for a given range width, the solutions may differ if the bounds of the ranges are different. For instance in table 4.2a, the bounds for J=2 differ from these for J=3. The results for these two cases are also different. Moreover, in table 4.2a for J=2 the solution is infeasible when the range width is 0.0313. However, for J=3 the solution is feasible for the same range width. This suggests that in cases where an acceptable solution is found, it may be possible to find an improved solution by using another set of promotion rate ranges of the same range width, and that in cases where, for a given range width, no feasible solution is found it may be possible to find an acceptable feasible solution by specifying other ranges with this range width. The strategy for searching for an improved solution is discussed in sections 4.3.1, 4.3.2 and 4.3.3.

4.3.1 Use of Overlapping Ranges

Suppose that at iteration k, $k \geq 1$, an acceptable solution, i.e. an optimal solution in which the promotion rate range width is acceptable to the decision makers, has been found, and that an improved solution with the same range width is desired. In order to search for an improved solution of this form, it is first necessary to choose a set of promotion rate ranges, and a procedure involving the use of overlapping ranges is now described. The same procedure can also be used when, for a specified range width, no feasible solution has been found, and a feasible solution of this range width is desired.

If an acceptable solution has been found at iteration k, the overall range of these overlapping promotion rate ranges for grade i is based on the optimal range for grade i at iteration k. If no feasible solution has been found at iteration k, the overall range of the overlapping promotion rate ranges for grade i is based on the optimal range for grade i at iteration k-1. To reduce the possibility of finding a local optimal, the overall range should extend beyond the appropriate optimal range. This overall range is then divided into J^* ranges each of range width equal to the range width at iteration k, such that range j, $1 \leq j < J^*$, overlaps range j+1, i.e. the upper bound of range j is an interior point of range j+1. These ranges will be referred to as overlapping ranges. The MIP model is then solved for this set of overlapping ranges. If an acceptable improved solution is found, then the process stops; otherwise another set of overlapping ranges of the same width is chosen, e.g. by increasing the value of J^* or using a different overall range, and the revised model is then solved. This process is repeated until an acceptable improved solution is found or until it is felt that the computational effort in searching for a feasible solution with the specified acceptable range width cannot be justified. A demonstration of overlapping ranges is presented in figure 4.1.

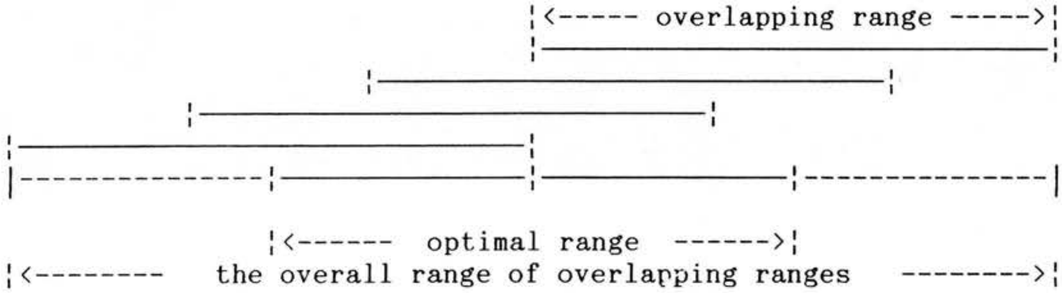


Figure 4.1 Demonstration of Overlapping Ranges

In order to use overlapping ranges in this way to search for an improved solution, the overall range must be specified. Note that the choice of overall range and associated bound values will affect the solution, i.e. the choice of overall range and the number of overlapping ranges will affect the solution. A method for selecting the bounds of the overlapping ranges is now described.

4.3.2 Specifying the Bounds of Overlapping Ranges

For a specified range width, it is assumed that the optimal range of the global optimal solution of this range width is near the optimal range found by the iterative procedure described in section 3.5. Assume that an acceptable solution is found at iteration k, k≥1. The optimal range for grade i at iteration k is used as a base for specifying the overall range of the overlapping ranges used in the search for an improved solution. This overall range is obtained by extending the optimal range at iteration k to the left and right by a specified proportion, defined as extension factor, Q^*, in order to increase the possibility of finding the global optimal solution. If no feasible solution has been found at iteration k, k>1, the overall range of overlapping ranges for grade i is obtained by extending the optimal range for grade i at iteration k-1 to the left and right by a specified proportion. Let

J^* number of the overlapping ranges for promotion rate, p_{it}, in grade i, i=1,2,...,I-1, in any year t, t=1,2,...T

$U_i^{(k)}$ upper bound of the optimal range for grade i, i=1,2,..,I-1, at iteration k, k=1,2,...

$L_i^{(k)}$ lower bound of the optimal range for grade i, i=1,2,...,I-1, at iteration k, k=1,2,...

Q^* extension factor, as a proportion of the optimal range used as the base for the overall range of overlapping ranges

$H^{(k)}$ the promotion rate range width for each grade at iteration k (assumed to be acceptable to the decision makers), i.e. the range width of the overlapping ranges, where $H^{(k)}=U_i^{(k)}-L_i^{(k)}$, k≥1, or if no feasible solution has been found at iteration k, k>1, $H^{(k)}=QH^{(k-1)}$, where Q is the range width reduction factor between successive iterations using non-overlapping ranges

H the overall range of overlapping ranges for each grade

D separation of lower bounds of successive overlapping ranges for each grade

$B_{2j-1,i}$ lower bound of the jth overlapping range, $j=1,2,...,J^*$, for grade i, i=1,2,...,I-1

$B_{2j,i}$ upper bound of the jth overlapping range, $j=1,2,...,J^*$, for grade i, i=1,2,...,I-1

For grade i, i=1,2,...,I-1, the overlapping ranges are arranged so that the lower bound of range 1 is the lower bound of the overall range, i.e.

$L_i^{(k)}-Q^*H^{(k)}$, and the upper bound of range J^* is the upper bound of the overall range, i.e. $U_i^{(k)}+Q^*H^{(k)}$, with the lower bounds of successive ranges being equally separated. Thus for case where an acceptable solution has been found at iteration k,

$$H = H^{(k)} + 2Q^*H^{(k)} \tag{4-1}$$

$$D = (H - H^{(k)})/(J^* - 1)$$

$$\text{i.e.}\quad D = 2Q^*(U_i^{(k)} - L_i^{(k)})/(J^* - 1) \tag{4-2}$$

$$B_{2j-1,i} = L_i^{(k)} - Q^*(U_i^{(k)} - L_i^{(k)}) + (j-1)D \tag{4-3}$$

$$B_{2j,i} = B_{2j-1,i} + U_i^{(k)} - L_i^{(k)} \tag{4-4}$$

$j=1,2,...,J^*$, $i=1,2,...,I-1$, $k=1,2,...$

Similarly, when no feasible solution has been found at iteration k,

$$H = H^{(k-1)} + 2Q^*H^{(k-1)} \tag{4-5}$$

$$D = (H - H^{(k)})/(J^* - 1)$$

Since no feasible solution can be found at iteration k, $H^{(k)}$ is obtained by using $H^{(k)}=QH^{(k-1)}$. Therefore,

$$D = (1 + 2Q^* - Q)(U_i^{(k-1)} - L_i^{(k-1)})/(J^* - 1) \tag{4-6}$$

$$B_{2j-1,i} = L_i^{(k-1)} - Q^*(U_i^{(k-1)} - L_i^{(k-1)}) + (j-1)D_i \tag{4-7}$$

$$B_{2j,i} = B_{2j-1,i} + Q(U_i^{(k-1)} - L_i^{(k-1)}) \tag{4-8}$$

$j=1,2,...,J^*$, $i=1,2,...,I-1$, $k=2,3,...$

For example, assume that an acceptable solution has been found at iteration k; the lower bound and upper bound of the optimal range for grade i at iteration k are 0.25 and 0.34 respectively; the extension factor, Q^*, for extending the optimal range in order to increase the possibility of covering the global optimal solution, is 50% and four overlapping ranges, i.e. $J^*=4$, will be formed within the overall range of overlapping ranges. Then,

$D = 0.03$; $B_{1i} = 0.205$, $B_{2i} = 0.295$; $B_{3i} = 0.235$, $B_{4i} = 0.325$;
$B_{5i}=0.265$, $B_{6i} = 0.355$; $B_{7i} = 0.295$, $B_{8i} = 0.385$

The resulting overlapping ranges are shown in figure 4.2.

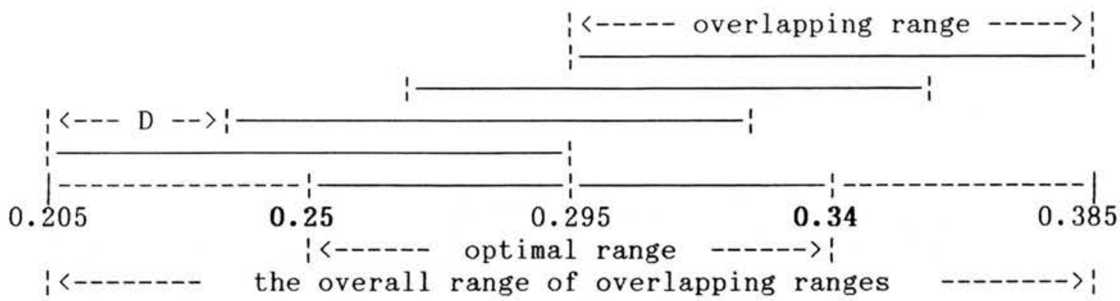


Figure 4.2 Example of Overlapping Ranges

A computer program has been written in BASIC to generate overlapping ranges in this way (see appendix C3).

4.3.3 Examples of Use of Overlapping Ranges

In this section some numerical experiments using the strategy of overlapping ranges will be conducted to attempt to find an improved solution. The effects on manpower cost and CPU time of the choice of values for the number of overlapping ranges, J^*, and the extension factor, Q^*, will be considered. The data used in this set of experiments are the same as in the numerical illustration of section 3.7.

From the results in table 4.2a it can be seen from the results for range width 0.0313 that in cases where no feasible solution is found, as in cases with 2 or 4 ranges, it may be possible to find a feasible solution by changing the bounds and/or the number of ranges. A systematic approach for attempting to find a feasible solution when no feasible solution has been found using the iterative procedure described in section 3.5 can be developed using overlapping ranges. For example, in table 4.2a at iteration (or run) 4, no feasible solution was found for the case with two ranges, i.e. $J=2$, of range width, $H^{(4)}=0.0313$. To obtain a feasible solution with this range width, the optimal range of the iteration prior to getting the infeasible solution, i.e. the optimal range at iteration 3 (see table 4.2b), is used as the overall range of the overlapping ranges. This overall range is divided into three overlapping ranges, i.e. $J^*=3$, with the same range width as for iteration 4. The MIP model was solved and a feasible solution found. The optimal solution is presented in table 4.4b. The impact of the number, J^*, of overlapping ranges on both the solution and the computational time was investigated by dividing the overall range into a J^* overlapping ranges, $J^*=4,5,\dots,10$, and solving the MIP model for each case. The results are presented in table 4.4a. From table 4.4a it can be seen that, in general, as the number of overlapping ranges increases the cost falls. The smallest and the largest costs in table 4.4a are £K2413536 and £K2429269, for $J^*=9$ and $J^*=3$ respectively. The optimal ranges for each case are presented in table 4.4b.

In cases where an acceptable solution is found, it may be possible to find an improved solution by using the strategy of overlapping ranges with the same range width. For example, in table 4.2a at iteration 4, when the optimal range in each grade is divided into three ranges, i.e. $J=3$, of range width $H^{(4)}=0.0313$, the solution is feasible. Assume that the decision makers accept this range width but want to know if there is a lower cost solution with this range width. In this case, the optimal range at iteration 4 (see table 4.2b) is used as the basis of the overall range of overlapping ranges. In order to increase the possibility of finding a lower cost solution, $Q^*=1$ is specified, i.e. the overall range of overlapping ranges extends to the left and right of the optimal range at iteration 4 by 100% respectively. The results obtained by dividing this overall range into J^* overlapping ranges, for $J^*=4,5,\ldots,10$, are presented in table 4.5a. The optimal ranges for these cases are presented in table 4.5b. The smallest and the largest costs in table 4.5a are £K2413685 and £K2429545, for $J^*=7$ and $J^*=5$ respectively. Note that the cost of every solution in table 4.5a is smaller than the cost in table 4.2a, £K2434893, for the same range width.

The results in tables 4.4a and 4.5a show that the larger the number of the overlapping ranges, J^*, the better the chance of finding a lower cost solution. The differences in the results are caused by assigning different bounds to the overlapping ranges. However, a larger value of J^* will increase the CPU time dramatically. For instance, in table 4.4a, as the number of the overlapping ranges, J^*, increases by a factor of 2.5, from $J^*=4$ to $J^*=10$, the CPU time for the Branch-and-Bound phase of the solution procedure increases by a factor of 69.4, from 62 seconds to 4305 seconds. The choice of the number of the overlapping ranges will depend on the level of accuracy sought by the decision makers. However, the results of the experiments suggest that only a small reduction in the cost of the solution is found by increasing the number of the overlapping ranges. For example, the percentage difference between the largest and the smallest costs in table 4.4a is only 0.65%.

The difference between the cost of the solution found by using the iterative solution procedure as described in section 3.5 and the solution found using the strategy of overlapping ranges is small. For example, in table 4.2a for three possible ranges the minimum cost obtained by using the basic iterative solution procedure at iteration 4, i.e. with range width 0.0313, is £K2434893. In table 4.5a, using the strategy of overlapping ranges with the same range width, the lowest cost is £K2413685. The percentage difference between these two costs is only 0.87%. In practice it may therefore not be necessary to use the overlapping range procedure, i.e. the iterative solution procedure described in section 3.5 may be regarded as satisfactory.

4.4 FACTORS AFFECTING THE COMPUTATIONAL TIME

The MIP manpower planning model was set up and solved using XPRESS-MP, a PC based MIP software product which utilises the branch and bound algorithm of integer programming, on a 386SX-16 laptop PC. It has been noted (e.g. Ashford and Daniel, 1992) that the effectiveness of computation using the branch and bound approach depends critically upon good model formulation, control of the branch and bound strategy, and the use of high level branching methods. A good model formulation is one in which the solution to the LP relaxation is as close as possible to the MIP solution. An important factor in a good formulation is the choice of a suitable value for M, as in constraints (3-18l) and (3-18m). The branch and bound solution strategies can be controlled by specification of the priorities of variables, choice of branching direction and the cut-off used for pruning the search tree. High level branching methods include the use of special ordered sets. In this section the effect on computational time of the value of M, variable priorities, and the use of special ordered sets will be investigated. The numerical experiments use the same data as in the numerical illustration of section 3.7 and five overlapping ranges as specified in table 4.6 for promotion rate, p_{it}, in each grade i in any year t.

4.4.1 Choice of Value of M

In all our previous numerical illustrations, the value of M in constraints (3-181) and (3-18m) was arbitrarily set to 4000. Camm, Raturi and Tsubakitani (1990), however, suggest that M should be made as small as possible in modelling a fixed charge problem, since computation times increase as larger values of M are used. The smallest M values for use in our experiments can be estimated from equation (3-17). From table 3.4 and figure 3.1 the maximum number of staff in any grade in any year is 2800, and from table 4.6 the maximum difference between the lower bounds of the first range and fifth range in each grade is 0.0626, and so from (3-17) M is given by M=0.0626*2800=175. A number of experiments were conducted in which values from 175 to 100000 were assigned to M in the MIP model constraints (3-181) and (3-18m). The computational times for the linear relaxation phase (i.e. LP phase) and for the branch and bound (BB) phase for each of these values of M are presented in table 4.7. From these results it can be seen that the smallest CPU time for the branch and bound phase is 122 seconds for M=1500. It can also be seen that for M values above 10000, the CPU times are stable and less than the CPU times with M values below 1000; for small values of M, for instance M=175 or 300, the CPU times in the branch and bound phase are much higher than when M is large, say 100000.

Another set of experiments was conducted using the narrower overall range of overlapping ranges presented in table 4.8, where the maximum difference between the lower bounds of the first and fifth range in each grade is 0.0157. In this case, the smallest M value is 44, i.e. M=0.0157*2800. The results for different M values are presented in table 4.9. Table 4.9 shows that for M=44 the CPU time in the branch and bound phase is 3.7 times that for M=100000. These results suggest that for a problem of this type it is better to make M large rather than to cut it down to size, as suggested by Camm, Raturi and Tsubakitani (1990).

4.4.2 Choice of Priorities

The computational time for solving an integer programming problem is sensitive to the priority in which the integer variables are chosen for branching. In XPRESS-MP the branching priority of integer variables can be set by assigning a priority to each variable, with a low value meaning that the variable has a higher priority in selection for branching. In previous experiments, the default priorities in the XPRESS-MP software, i.e. 500, was used for every binary variable. In this section the effect on computational time of changing the variable priorities will be investigated using the same data as in the numerical illustration of section 3.7 and the five overlapping ranges specified in table 4.6, and with M=4000. The effect of the priorities of the binary variables will be investigated in three ways, namely in terms of grade, range, and the combination of grade and range.

Forrest, Hirst, and Tomlin (1974) suggest that the priorities in the branching process of the branch and bound algorithm should reflect the importance of the integer variable to which they are attached; the more important the variable, the higher its priority for selection in branching. The importance of a variable may be specified in several ways. A robust method for identifying such variables has not been established, but a common way of choosing a branching variable is by user specified priorities. In the MIP model for manpower planning, the promotion rate from grade i to i+1, i=1,2,3,4,5, has five ranges, and a binary variable is associated with each range, i.e. 25 binary variables in total. In the experiments the priorities of the binary variables were assigned in three ways: in terms of grade, range, and a combination of grade and range. By concentrating on grade, each range at the same grade gets the same priority. By concentrating on range, the priority of each promotion rate range is established first. In assigning the priority index on the combination of grade and range, the priorities of grades are decided first, then the priorities of promotion rates ranges within the grades are set. For example, if the priority of promotion from grade 1 to grade 2 is higher than grade 2 to grade 3 then the priorities of the binary variables associated with ranges for promotion from grade 1 to grade 2 are higher than the priorities of the binary variables associated with ranges for promotion from grade 2 to grade 3. Note that if variables have the same priority, they will be chosen first in accordance with the highest estimated cost by the XPRESS-MP optimiser (Dash Associates, 1991).

4.4.2.1 Grade Based Priorities

The computational times in the branch and bound phase for priorities based on grade are presented in table 4.10. Intuitively, it can be argued that promotion in higher grades is more important than in lower grades since the promotion in the lower grades is affected by the vacancies in the higher grades. This is the basis of experiment 1. The basis of experiment 2 is the opposite of experiment 1, i.e. promotion in lower grades is more important than in higher grades. In experiment 3, the most important promotion is assigned to the middle grade, i.e. promotion from grade 3 to grade 4. The CPU time for the experiments investigating the effect of priorities on computational time are presented in table 4.10. It can be seen that in the first three experiments, the CPU time in experiment 1 is smaller than experiments 2 and 3.

The priorities used in experiment 4 are the same as in experiment 1, but with the priorities for promotion from grades 4 and 5 reversed, i.e. grade 4 is assigned the higher priority in the branching process. The CPU time in experiment 4 is smaller than in experiment 1 (table 4.10), although the difference is small. This suggests that the highest priority should be assigned to promotion from grade 4 to grade 5 and the second priority should be assigned to promotion from grade 5 to grade 6. By fixing these two priorities, six sets of priorities can be obtained, as considered experiments 4 to 9. In these nine experiments, the smallest two CPU times, 115 and 120 seconds, were shown in experiments 8 and 9. In experiments 10 and 11 the priorities of grades 4 and 5 the reverse of those in experiments 8 and 9 respectively, and in each case the CPU time was greater than in the associated earlier experiment. From table 4.10 it can be seen that experiment 8, i.e. with priority set (3,4,5,1,2), has the smallest CPU time.

4.4.2.2 Range Based Priorities

The results for experiments in which the priorities are based on range are presented in table 4.11, and show that the priorities strongly influence computational time. The smallest CPU time for the branch and bound phase is 295 seconds, for the priority sets (4,5,3,1,2) and (4,5,3,2,1), i.e. experiments 18 and 22 respectively, while the greatest CPU time is 1676 seconds for priority set (5,3,1,2,4), i.e. experiment 15. Note that the CPU times in this set of experiments are generally higher than the CPU times in the experiments using grade based priorities.

4.4.2.3 Combination of Grade and Range Based Priorities

The number of possible sets of priorities is large, and therefore only a limited number of priority assignments can be investigated. However, the CPU times in tables 4.10 and 4.11 indicate that grade plays a more important role than range in the branch and bound phase of the solution of the MIP model of a manpower system. Therefore, in considering priorities based on the combination of grade and range, the best priority sets in tables 4.10 and 4.11, i.e. experiments 8, 18 and 22, are used as the basis of the combination of the grade and range experiments. The results of these experiments are shown in table 4.12. The CPU times in experiments 23 and 24 are higher than in experiment 8. This suggests that it is better to give ranges within a grade the same priorities, as in experiment 8, than to assign separate priorities to each range, as in experiments 23 and 24. In experiment 25, ranges 4 and 5 are assigned the same priorities, while in experiment 26, ranges 1,3,4 and 5 are assigned the same priorities. In experiment 27, ranges 4 and 5, and range 1,2 and 3 are assigned the same priorities and this priority set yields the smallest CPU time for the branch and bound phase, 104 seconds, in the complete set of experiments.

Note that in the set of experiments investigating the effect of changing the priorities of variables, the value of M was 4000, while in the experiments involving different M values (see table 4.7), the smallest CPU time in the branch and bound phase is 122 seconds for M=1500. However, when the value of M in the priority set found to produce the smallest CPU time in the branch and bound phase, i.e. as in experiment 27, was reduced from 4000 to 1500, the CPU time in the branch and bound phase was 124 seconds, i.e. higher than the CPU time of 104 seconds for M=4000. This shows that the effect of the value of M and the assignment of variable priorities on computational time is complex, and that in particular using the best value for M with the default priority settings does not yield the lowest computational time.

4.4.3 Special Ordered Sets

There are two types of special ordered sets - special ordered sets of type one (SOS1) and special ordered sets of type two (SOS2). An SOS1 is a set of variables of which only one can take on a non-zero value. An SOS2 is a set of variables in which at most two adjacent variables can be non-zero. The MIP model of a manpower system contains special ordered sets of type 1 since only one of the ranges for promotion from grade i to i+1 can be chosen.

In an SOS1, the binary variables r_j, j=1,2,...,n, are such that

$$r_1 + r_2 + \ldots + r_n = 1$$

To use an SOS1 in the branch and bound algorithm, a weight W_j is assigned to variable r_j where the weights W_j are monotonic, and a variable w is defined by

$$w = W_1 r_1 + W_2 r_2 + \ldots + W_n r_n$$

If none of the variables r_j is equal to 1, the value of w is used in the selection of the branching variable. For example, if

$$W_r < w \le W_{r+1}$$

then branching on the SOS can be performed by imposing the requirement that either $w \geq W_{r+1}$

or $w \leq W_r$

This branching can be performed by imposing the requirement that

either $r_1 = r_2 = r_3 = \ldots = r_r = 0$

or $r_{r+1} = r_{r+2} = \ldots = r_n = 0$

Special ordered sets can be a powerful device for reducing computational time in solving MIP models. Some examples of the use of special ordered sets can be found in Forrest, Hirst and Tomlin (1974), and Creegan and Monforte (1990). The data used in this section is the same as in the numerical illustration of section 3.7 and in table 4.6 with M=4000. The effect on computational time of changing the weights of the variables associated with the sets will be investigated.

In the MIP model (3-18), five special ordered sets of type 1 are required, set i, i=1,2,3,4,5, being associated with promotion from grade i to i+1. For the case where five promotion rate ranges are considered, each set consists of five binary variables. For convenience, each special ordered set is given the same weight. A group of numerical experiments were performed in which different weights were assigned to the binary variables associated with the promotion rate ranges. The results of these experiments are shown in table 4.13. All the CPU times in the branch and bound phase of table 4.13 are greater than the time obtained by using the default priority with M=4000 and without using the SOS1 facility in XPRESS-MP, i.e. 147 seconds in table 4.7. In these cases the use of special ordered sets has not reduced the computational time. Since the efficiency of processing of special ordered sets depends on the weights given to the variables within the sets, it may be that the weights assigned to the binary variables were not suitable.

4.5 SUMMARY

In Chapter 3 an MIP model of a manpower system was developed for a non-linear manpower planning problem. The essential features in the model involve treating promotion rates as variables and imposing constraints to keep promotion rates as stable as possible from year to year. The model is solved using an iterative procedure in which the promotion rate range width is reduced at successive iterations until a solution which is either acceptable in terms of the range of variation in promotion rates, or infeasible is obtained.

Since the solution of the MIP model is influenced by the number, J, of the promotion rate ranges, and the reduction factor, Q, i.e. the factor by which the range width is reduced between successive iterations, it is desirable to determine suitable values for J and Q. In this chapter the effect on the solution of the choice of values for the number of promotion rate ranges and the reduction factor has been considered by conducting some numerical experiments. When an solution in which the promotion rate range width is acceptable to the decision makers has been found using the iterative solution procedure described in section 3.5, it may be possible to find a lower cost solution with the same range width. In cases where no acceptable feasible solution has been found, it may be possible to find a feasible solution with the acceptable range width. In this chapter a solution strategy involving the use of overlapping ranges has been developed to deal with these possibilities. It has also been noted that the computational effort in solving the model will depend on the factors such as the value of M in constraints (3-18l) and (3-18m), the priority of integer variables chosen for branching, and the use of special ordered sets. The effect of these factors on the CPU time has also been considered in this chapter.

The results of the numerical experiments for the choice of values for the number of ranges, J, and the reduction factor, Q, show the following.

1. A suitable choice of value for J is either 2 or 3.

2. There is no evidence to show that increasing the overlap values, i.e. the amount by which the overall range at iteration k+1 overlaps ranges adjacent to the optimal range at iteration k, will yield a lower cost solution. This means that for a given range width, increasing the number of ranges does not guarantee a better solution.

3. By adding equal width ranges adjacent to the optimal range and solving the problem again, the solution does not change. This indicates that the iterative solution procedure converges for this problem. It also implies that for a specified range width, the global optimal solution is near the optimal range generated by using the iterative solution procedure.

4. If an acceptable solution, in terms of the range of variation in promotion rates, is found, it may be possible to find a lower cost solution, and in cases where no acceptable feasible solution is found it may be possible to find a feasible solution with an acceptable range width by specifying other sets of ranges for promotion rates.

In the numerical experiments based on the use of overlapping ranges, it was found that for a specified range width, the differences in the cost of the solutions obtained by using the basic iterative solution procedure described in section 3.5 and the overlapping ranges strategy are small. In these experiments the percentage difference between these two costs was only 0.87%. This indicates that the optimal solution obtained by using the basic iterative solution procedure is generally satisfactory.

For the MIP based manpower planning model for a specified number of periods, the computational effort in solving the problem will depend on the value of M in constraints (3-18l) and (3-18m), the priority of integer variables chosen for branching and the use of special ordered sets. The numerical experiments show that for this MIP model it is better to make M large rather than to cut it down in size, as suggested by Camm, Raturi and Tsubakitani (1990). The CPU times in the branch and bound phase using branching priorities based on grade are smaller than those based on range. The results also indicate that the value of M and the branching priorities should be chosen jointly to obtain the most appropriate combination for a particular problem. In practice, experiments should be carried out to find appropriate settings for a particular class of problem.

In order to simplify the model, age and length of service have not been considered in this model of a manpower system. However, age and length of service have an important role to play in manpower planning. When a recruitment bulge reaches retirement age, an excessive demand for recruits to fill the vacancies will lead to a new recruitment bulge and so perpetuate the problem. The erratic vacancies caused by irregular retirement will influence the promotion opportunities, i.e. individuals will be lucky or unfortunate in their promotion opportunities because of excessive vacancies or bottlenecks. Although the number of recruits and the promotion rate stability constraints in the MIP model, i.e. constraints (3-18j), (3-18k), (3-18l), (3-18m) and (3-18n), have been introduced to alleviate these problems, the ranges for recruitment and promotion may be not sufficiently narrow. For example, a narrower promotion rate range may conflict with the other constraints, such as the target number of staff in each grade, i.e. constraints (3-18h) and (3-18i). Moreover, length of service is an important qualification when considering promotion in some organisations, such as the police or the armed forces. Different promotion and retirement policies in terms of length of service and retirement age will affect the career prospects of recruits and ultimately the manpower supply for the organisation. It is therefore desirable to take account of age and length of service distributions in the model of the manpower system. A model which provides a means for evaluating the effects of changing retirement age and takes account of length of service will be developed in Chapter 5.

CHAPTER 5

A MODEL INVOLVING GRADE, SERVICE LENGTH AND AGE

5.1 INTRODUCTION

The MIP model has been developed to determine the minimum cost recruitment and promotion strategies that will satisfy the manpower needs while ensuring promotion rates remain stable over time. The model is solved using an iterative procedure in which the promotion rates range width is reduced at successive iterations, until either an infeasible solution is found or the range of promotion rate variability is acceptable to the decision makers. In the model, age and total length of service, defined as the years of service from the first day of entering the system, have been ignored. However, in some systems such as the police and the armed forces, total length of service is an important qualification for promotion. Changes in policies regarding age and total length of service required for retirement and promotion will have cost implications and also cause problems in the demand for and supply of manpower, and the career opportunities of individuals. These career opportunities are concerned with promoting individuals at the appropriate stage in their development and eliminating disparities in promotion opportunities. A model based on the MIP model described in Chapter 3 and involving age and total length of service in each grade is now developed.

5.2 MODEL DESCRIPTION AND NOTATION

The model is developed for an organisation in which the only form of promotion involves transfer to the next higher grade, with staff considered for promotion only after completing a required minimum total length of service in that grade. It is assumed that recruitment occurs only in the lowest grade, and that wastage and retirement occur in all grades. It is also assumed that the total length of service for all staff in grade i at retirement is greater than the minimum total length of service required for promotion from grade i, $i=1,2,\ldots,I-1$. The lowest grade, grade 1, obtains staff only by recruitment and the highest grade, grade I, loses staff only by wastage and retirement. Demotions from any grade are not allowed. Staff wastage results from ill-health, death and those who leave the organisation of their own choice, while retirement refers to those who leave the organisation on reaching retirement age. It is assumed that the retirement age is a non-decreasing function of grade. It is also assumed that recruitment occurs only in the lowest grade and within a narrow age range and that the age distribution of recruits is stable over time. This corresponds to the situation in many military systems.

The costs considered in the model consist of recruitment costs, stock costs and pension costs. The pension costs consist of a lump sum payment on leaving the system and the discounted cost of the annual pension. Individuals who leave with a total length of service which is less than the required minimum total length of service for annual pension entitlement, receive a single lump sum payment, which depends on grade and length of service, on leaving the system. It is assumed that the total length of service for all staff in grade i, $i=1,2,\ldots,I$, at retirement age is greater than or equal to the required minimum total length of service for annual pension entitlement. Pension payments, which depend on grade and length of service on leaving the system, are made annually and adjusted in accordance with inflation until death. The pension costs will be affected by the policies on retirement age and the required minimum total length of service for annual pension entitlement. In developing the model, it is assumed that the lump sum payments and annual pensions are paid only to individuals who have satisfied the required service conditions, and that no payments are made to dependent relatives, either as a lump sum following death in service or as a reduced level pension following the death of the pensioner. The model can, however, be extended for the case where payments are made to surviving dependent relatives.

5.2.1 Notation

In the model, as in the MIP model described in Chapter 3, year t is defined from time t-1 to time t, where t is an integer. The number of staff in grade i with total length of service h and of age e at the end of year t is a stock, defined as n_{ihet}. The number that move from grade i to i+1, and the numbers entering or leaving the system in year t, i.e. from time t-1 to t, are flows. These flows consist of recruitment, promotion, wastage and retirement. For simplicity, all these flows are presumed to occur at the end of year t. The rates of promotion and wastage of staff in grade i with total length of service h and of age e at the end of year t, p_{ihet} and W_{ihet} respectively, are defined as proportions of the number of staff in grade i with total length of service h-1 and of age e-1 at the end of year t-1. The number of promotions of staff with total length of service h and of age e at the end of year t from grade i to i+1 is given by $m_{ihet}=p_{ihet}n_{i,h-1,e-1,t-1}$.

5.2.2 Model Parameters

The coefficients in the MIP model are defined as follow:

- A_L minimum age of recruitment in grade 1
- A_U maximum age of recruitment in grade 1
- A_E maximum life expectancy
- D_e the proportion of recruits of age e, $e=A_L,A_L+1,...,A_U$, in the total number of recruits in any year
- A_{Ri} retirement age in grade i, $i=1,2,...,I$
- H_{Pi} required minimum total length of service for promotion from grade i to i+1, $i=1,2,...,I-1$
- H_{Li} minimum total length of service of staff in grade i, $i=1,2,...,I$, equal to the required minimum total length of service for promotion from grade i-1 to i, i.e. $H_{Li}=H_{Pi-1}$, where $H_{L1}=0$
- H_R required minimum total length of service for annual pension entitlement
- H_{Ui} maximum total length of service in grade i at retirement, given by $H_{Ui}=A_{Ri}-A_L$, $i=1,2,...,I$
- N_{ihe0} number of staff in grade i, $i=1,2,...,I$, with total length of service h, $h=H_{Li},H_{Li}+1,...,H_{Ui}$, and of age e, $e=A_L+h,A_L+h+1,...,\mathrm{Min}\{A_U+h,A_{Ri}\}$, initially, i.e. at start of year 1 (or end of year 0)
- W_{ihet} the wastage rate in grade i, $i=1,2,...,I$, of staff with total length of service h, $h=H_{Li}+1,H_{Li}+2,...,H_{Ui}$, and of age e, $e=A_L+h,A_L+h+1,...,\mathrm{Min}\{A_U+h,A_{Ri}\}$, at the end of year t,

$t=1,2,...,T$, as a proportion of the number of staff in grade i with total length of service h-1 and of age e-1 at the end of year t-1

W'_{ihet} the wastage rate for whatever reason except death of individuals in grade i, $i=1,2,...,I$, of staff with total length of service h, $h=H_{Li}+1,H_{Li}+2,...,H_{Ui}$, and of age e, $e=A_L+h,A_L+h+1,...,Min\{A_U+h,A_{Ri}\}$, at the end of year t, $t=1,2,...,T$

S_t target total number of staff in the manpower system at the end of year t, $t=1,2,...,T$

E_{Ut} maximum upper proportional deviation in target total number of staff at the end of year t, $t=1,2,...,T$

E_{Lt} maximum lower proportional deviation in target total number of staff at the end of year t, $t=1,2,...,T$

F_{Uit} maximum upper proportional deviation in target number of staff in grade i, $i=1,2,...,I$, at the end of year t, $t=1,2,...,T$

F_{Lit} maximum lower proportional deviation of target number of staff in grade i, $i=1,2,...,I$, at the end of year t, $t=1,2,...,T$

G_{it} target proportion of staff in grade i, $i=1,2,...,I$, at the end of year t, $t=1,2,...,T$

R_{Ut} upper bound on the number of recruits to grade 1 at the end of year t, $t=1,2,...,T$

R_{Lt} lower bound on the number of recruits to grade 1 at the end of year t, $t=1,2,...,T$

$B^{(k)}_{ji}$ lower bound of promotion rate range j, $j=1,2,...,J$, from grade i to grade i+1, $i=1,2,...,I-1$, at iteration k, $k=1,2,...,$ for all staff in grade i in any year

$B^{(k)}_{j+1,i}$ upper bound of promotion rate range j, j=1,2,...,J, from grade i to grade i+1, i=1,2,...,I-1, at iteration k, k=1,2,..., for all staff in grade i in any year

C_{Rt} average recruitment cost per person recruited to grade 1 in year t, t=1,2,...,T

C_{Siht} average annual salary per person in grade i, i=1,2,...,I, with total length of service h, $h=H_{Li},H_{Li}+1,...,H_{Ui}$, in year t, t=1,2,...,T

C_{Fiht} average lump sum payment per person for those who leave the system in grade i, i=1,2,...,I, with total length of service h, $h=H_{Li},H_{Li}+1,...,H_{Ui}$, in year t, t=1,2,...,T

C_{Piht} average annual pension per person for those who leave the system in grade i, i=1,2,...,I, with total length of service h, $h=H_{Li},H_{Li}+1,...,H_{Ui}$, in year t, t=1,2,...,T, pension is paid annually after leaving the system, and adjusted in accordance with annual rate of increase of salary, ß, until death

$Y_{e,e+z}$ the probability of survival from age e to age e+z, $e=A_L+H_R,A_L+H_R+1,...,A_{Ri}$, $z=1,2,...,A_E-e$, i=1,2,...,I

α annual discount rate

β annual rate of increase of salary for inflation

5.2.3 Variables:

The variables in the MIP model are defined as below:

n_{ihet} number of staff in grade i, i=1,2,...,I, with total length of service h, $h=H_{Li},H_{Li}+1,...,H_{Ui}$, and of age e, $e=A_L+h,A_L+h+1,...,\mathrm{Min}\{A_U+h,A_{Ri}\}$ at the end of year t, t=1,2,...,T

s_t the total number of staff in the manpower system at the end of year t, $t=1,2,\ldots,T$

r_t number of recruits to grade 1 at the end of year t, $t=1,2,\ldots,T$

p_{ihet} promotion rate from grade i to i+1, $i=1,2,\ldots,I-1$, for staff with total length of service h, $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$, and of age e, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$ at the end of year t, $t=1,2,\ldots,T$, as proportion of staff in grade i with total length of service h-1 and of age e-1 at the end of year t-1

m_{ihet} number of staff promoted from grade i to i+1, $i=1,2,\ldots,I-1$, with total length of service h, $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$ and of age e, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$ at the end of year t, $t=1,2,\ldots,T$, i.e. $m_{ihet}=p_{ihet}n_{i,h-1,e-1,t-1}$

δ_{ji} binary variable of promotion rate range j, $j=1,2,\ldots,J$, from grade i, $i=1,2,\ldots,I-1$, to grade i+1, where $\delta_{ji}=1$ if the jth range is chosen; otherwise, $\delta_{ji}=0$

5.2.4 Objective Function

The objective function involves the minimisation of total manpower costs. These costs consist of recruitment costs, stock costs, and pension costs. The recruitment costs include the costs incurred in the process of recruitment and the costs of training of recruits before they become fully operational. The recruitment costs in year t are the product of the average recruitment cost per person in year t, C_{Rt}, and the number of recruits in year t, r_t. The stock cost in grade i for staff with total length of service h in year t is the product of the average annual salary per person, C_{Siht}, and the average number of staff in grade i with total length of service at end of year t. Since all changes are assumed to occur at the end of a year, the average number of staff in year t is the number of staff at the end of year t. The minimum and maximum ages for recruitment, and the retirement age in grade i are defined as A_L, A_U, and A_{Ri} respectively. Note that the maximum total length of service in grade i before retirement is $H_{Ui}-1$, where H_{Ui} is the maximum total length of service for staff in grade i. Clearly, H_{Ui} is given by subtracting A_L from A_{Ri}, and the range of the ages associated with each total length of service h in grade i before retirement is A_L+h to $\mathrm{Min}\{A_U+h, A_{Ri}-1\}$.

The pension costs involve the discounted costs of lump sum payments and annual pension payments. Those who serve less than the required minimum total length of service for annual pension entitlement, H_R, will get a single lump sum payment, C_{Fiht}, which depends on grade i and total length of service h, on leaving the system; otherwise the annual pension, C_{Piht}, will be paid annually in the subsequent years from leaving the system and adjusted in accordance with the rate of increase of salary, ß, until death. The range of the number of years for receiving annual pension is from 1 to A_E-e, where A_E is the maximum life expectancy and e is the age of the staff leaving the system. It is assumed that dependent relatives of those who die in service or after retirement do not receive either a lump sum payment or an annual pension at a reduced level, and W'_{ihet} is defined as the wastages from all causes except death of individuals in grade i with total length of service h and of age e at the end of year t.

The objective function for the model is outlined below.

Minimise

$$\sum_{t=1}^{T} C_{Rt}(1+\alpha)^{-t} r_t + \sum_{i=1}^{I} \sum_{t=1}^{T} \{ \sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h,A_{Ri}-1\}} C_{Siht}(1+\alpha)^{-t} n_{ihet} +$$

$$\sum_{h=H_{Li}+1}^{H_R-1} \sum_{e=A_L+h}^{A_U+h} C_{Fiht}(1+\alpha)^{-t}(W'_{ihet} n_{i,h-1,e-1,t-1}) +$$

$$\sum_{h=H_R}^{H_{Ui}} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h,A_{Ri}\}} \sum_{z=1}^{A_E-e} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} W'_{ihet} n_{i,h-1,e-1,t-1}$$

$$\sum_{h=A_{Ri}-A_U}^{H_{Ui}} \sum_{e=A_{Ri}}^{A_{Ri}} \sum_{z=1}^{A_E-A_{Ri}} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} n_{ihet} \} \qquad (5\text{-}1)$$

Note that the costs of the annual pension of those leaving in year t is the sum of payments in all subsequent years until death. Note also that all the costs are discounted to the year 0. Assume that those who are entitled to the annual pension leave the system at age e. Note that the probability of survival from age e to age e+z, $Y_{e,e+z}$, is given by

$$Y_{e,e+z} = Y_{e,e+1} Y_{e+1,e+2} \cdots Y_{e+z-1,e+z}$$

and is calculated recursively, i.e.

$$Y_{e,e+z} = Y_{e,e+z-1} Y_{e+z-1,e+z}$$

with $Y_{e,e}=1$, $e=A_L+H_R, A_L+H_R+1, \ldots, A_{Ri}$, $z=1,2,\ldots,A_E-e$, where A_L, H_R and A_E are the minimum age at recruitment, the required minimum total length of service for annual pension entitlement, and the maximum life expectancy respectively.

5.2.5 Constraints

5.2.5.1 Manpower Stocks

The number of staff in grade i with total length of service h and of age e at the end of year t, n_{ihet}, is equal to the number of staff in grade i with total length of service h-1 and of age e-1 at the end of year t-1, i.e. $n_{i,h-1,e-1,t-1}$, less wastage and promotions from grade i, plus promotions to grade i at the end of year t. Since recruitment only occurs in grade 1, the number of staff of age e entering the system at the end of year t, i.e. n_{10et}, is the product of the proportion, D_e, of recruits of age e in the total number of recruits and the number of recruits at the end of year t, r_t. Note that promotion from each grade i occurs only on completion of a specified minimum number of years of service, i.e. there is no promotion of those staff in grade i with total length of service h less than the required minimum total length of service H_{Pi} for promotion from grade i to i+1. Clearly, the maximum age associated with the required minimum total length of service for promotion from grade i, i.e. A_U+H_{Pi}, must be less than the retirement age in grade i, A_{Ri}. Note that the minimum total length of service in grade i, H_{Li}, is equal to the required minimum total length of service for promotion from grade i-1 to i, H_{Pi-1}. When the total length of service h in grade i is equal to the minimum total length of service in grade i, i.e. $h=H_{Li}$, the number of staff in grade i for $h=H_{Li}$ is equal to the number of staff promoted from grade i-1 to i with that total length of service, i.e. $n_{ihet}=m_{i-1,het}$ as $h=H_{Li}$, $i=2,3,\ldots,I$. The manpower stocks constraints derived from the MIP model (3.18), i.e. the constraints (3-18b) to (3-18d) become:

$$n_{10et}-D_e r_t=0 \qquad (5\text{-}2a)$$

$e=A_L,A_L+1,\ldots,A_U,\ \forall t$

$$n_{1het}-(1-W_{1het})n_{1,h-1,e-1,t-1}=0 \qquad (5\text{-}2b)$$

$h=1,2,\ldots,H_{P1}-1,\ e=A_L+h,A_L+h+1,\ldots,A_U+h,\ \forall t$

$$n_{1het}-(1-W_{1het})n_{1,h-1,e-1,t-1}+m_{1het}=0 \qquad (5\text{-}2c)$$

$h=H_{P1},H_{P1}+1,\ldots,H_{U1},\ e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{R1}\},\ \forall t$

$$n_{Ihet}-(1-W_{Ihet})n_{I,h-1,e-1,t-1}-m_{I-1,het}=0 \quad (5\text{-}2d)$$

$h=H_{LI}+1,H_{LI}+2,\ldots,H_{UI},\ e=A_L+h,A_L+h+1,\ldots,Min\{A_U+h,A_{RI}\},\ \forall t$

$$n_{ihet}-m_{i-1,het}=0 \quad (5\text{-}2e)$$

$i=2,3,\ldots,I,\ h=H_{Li},\ e=A_L+h,A_L+h+1,\ldots,A_U+h,\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-m_{i-1,het}=0 \quad (5\text{-}2f)$$

$i=2,3,\ldots,I-1,\ h=H_{Li}+1,H_{Li}+2,\ldots,H_{Pi}-1,\ e=A_L+h,A_L+h+1,\ldots,A_U+h,\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-m_{i-1,het}+m_{ihet}=0 \quad (5\text{-}2g)$$

$i=2,3,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ e=A_L+h,A_L+h+1,\ldots,Min\{A_U+h,A_{Ri}\},\ \forall t$

Note that in constraints (5-2d) to (5-2g), the number of promotions from grade i-1 to grade i with total length of service h and of age e at the end of year t, $m_{i-1,het}$, does not exist when the value of total length of service h exceeds the maximum total length of service in grade i-1, H_{Ui-1}. Since it is assumed that the retirement age at grade i is greater than or equal to that at grade i-1, $i=2,3,\ldots,I$, the maximum total length of service in grade i at retirement is greater than or equal to that in grade i-1, i.e. $H_{Ui} \geq H_{Ui-1}$. Therefore, we must let

$$m_{i-1,het}=0 \quad (5\text{-}2h)$$

where $i=2,3,\ldots,I$, $h>H_{Ui-1}$, $e=A_L+h,A_L+h+1,\ldots,Min\{A_U+h,A_{Ri}\}$, $\forall t$. Alternatively, variables $m_{i-1,het}$ in constraints (5-2d) to (5-2g) can be multiplied by binary coefficients, $\theta_{i-1,h}$, to force variables $m_{i-1,het}$ to be zero when the value of h exceeds H_{Ui-1}, i.e. $\theta_{i-1,h}=0$ $i=2,3,\ldots,I$, $h>H_{Ui-1}$, and $\theta_{i-1,h}=1$ $i=2,3,\ldots,I$, $h \leq H_{Ui-1}$. Therefore, constraints (5-2d) to (5-2g) are revised as below.

$$n_{Ihet}-(1-W_{Ihet})n_{I,h-1,e-1,t-1}-\theta_{I-1,h}m_{I-1,het}=0 \quad (5\text{-}2d1)$$

$h=H_{LI}+1,H_{LI}+2,\ldots,H_{UI},\ e=A_L+h,A_L+h+1,\ldots,Min\{A_U+h,A_{RI}\},\ \theta_{I-1,h}=0 \text{ for } h>H_{UI-1},\ \theta_{I-1,h}=1 \text{ for } h\leq H_{UI-1},\ \forall t$

$$n_{ihet}-\theta_{i-1,h}m_{i-1,het}=0 \quad (5\text{-}2e1)$$

$i=2,3,\ldots,I,\ h=H_{Li},\ e=A_L+h,A_L+h+1,\ldots,A_U+h,\ \theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\leq H_{Ui-1},\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-\theta_{i-1,h}m_{i-1,het}=0 \quad (5\text{-}2f1)$$

$i=2,3,\ldots,I-1,\ h=H_{Li}+1,H_{Li}+2,\ldots,H_{Pi}-1,\ e=A_L+h,A_L+h+1,\ldots,A_U+h,$

$\theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-\theta_{i-1,h}m_{i-1,het}+m_{ihet}=0 \qquad (5\text{-}2g1)$$

$i=2,3,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\},$

$\theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$

5.2.5.2 The Total Staff in the System

The total staff, s_t, at the end of year t is the summation of the staff over all grades, service length and ages. The maximum age and total length of service before retirement in grade i are $A_{Ri}-1$ and $H_{Ui}-1$. For the sake of acceptability, the total staff must be within a range defined in terms of deviations for target total number of staff, S_t, for year t:

$$s_t-\sum_{i=1}^{I}\sum_{h=H_{Li}}^{H_{Ui}-1}\sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h,A_{Ri}-1\}} n_{ihet}=0 \qquad (5\text{-}3a)$$

$$s_t \ge S_t(1-E_{Lt}) \qquad \forall t \qquad (5\text{-}3b)$$

$$s_t \le S_t(1+E_{Ut}) \qquad \forall t \qquad (5\text{-}3c)$$

where E_{Lt} and E_{Ut} are the lower and upper proportional deviations in the target total number of staff in year t respectively. The range of proportional deviation in the target total number of staff can be altered to investigate the effect on the manpower system.

5.2.5.3 The Staff in Each Grade

The number of staff in each grade i at the end of year t is the summation over all service length and ages of staff in grade i at the end of year t. This number must be within a range defined in terms of deviations for target number of staff in grade i. The target number of staff in grade i at the end of year t is the product of the target proportion of staff in grade i at end of year t, G_{it}, and the target total number of staff in the manpower system at end of year t, S_t.

$$\sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{Min\{A_U+h, A_{Ri}-1\}} n_{ihet} \geq G_{it}S_t(1-F_{Lit}) \qquad \forall i,t \qquad (5\text{-}4a)$$

$$\sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{Min\{A_U+h, A_{Ri}-1\}} n_{ihet} \leq G_{it}S_t(1+F_{Uit}) \qquad \forall i,t \qquad (5\text{-}4b)$$

where F_{Lit} and F_{Uit} are the lower and upper proportional deviations in the target number of staff in grade i in year t. The parameter G_{it} in constraints (5-4) can be changed in order to observe the influence of different grade structures on the system.

5.2.5.4 The Number of Recruits

Recruitment only occurs in grade 1 at the end of each year t. To avoid an irregular age distribution caused by fluctuating recruitment in each year, the number of recruits must be within a defined range.

$$r_t \geq R_{Lt} \qquad \forall t \qquad (5\text{-}5a)$$

$$r_t \leq R_{Ut} \qquad \forall t \qquad (5\text{-}5b)$$

The lower bound and upper bound of the number of recruits, R_{Lt} and R_{Ut} respectively, are specified by the decision makers.

5.2.5.5 Stable Promotion Rates

In the model, promotion from grade i occurs only when the total length of service in grade i is greater than or equal to H_{Pi}, the required minimum total length of service for promotion from grade i. The requirement for p_{ihet}, the rate of promotion from grade i of staff with total length of service h and of age e at the end of year t, to be stable over time, can be modelled by imposing constraints to ensure that the promotion rate is in the same range in each year.

Assume that at iteration k in the iterative solution procedure, J ranges are used to define the possible ranges for promotion rate, p_{ihet}, in any year. The lower and upper bounds of these ranges are denoted by $B^{(k)}_{jihe}$ and $B^{(k)}_{j+1,ihe}$ respectively. Define δ_{jihe} as a binary variable, which equals one if jth range is selected and is zero otherwise, where $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$ $i=1,2,\ldots,I-1$, $j=1,2,\ldots,J$. The number of staff promoted from grade i with total length of service h and of age e at the end of year t is m_{ihet} and is given by

$$m_{ihet}=p_{ihet}n_{i,h-1,e-1,t-1}$$

where $i=1,2,\ldots,I-1$, $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$, $t=1,2,\ldots,T$.

Therefore, for stable promotion rates the required constraints derived from constraints (3-18l) to (3-18n) in the MIP model (3-18) are:

$$-m_{ihet} + B^{(k)}_{jihe}n_{i,h-1,e-1,t-1} + M\delta_{jihe} \le M \qquad (5\text{-}6a)$$

$$m_{ihet} - B^{(k)}_{j+1,ihe}n_{i,h-1,e-1,t-1} + M\delta_{jihe} \le M \qquad (5\text{-}6b)$$

$$\sum_{j=1}^{J} \delta_{jihe} = 1 \qquad (5\text{-}6c)$$

where M is a sufficiently large number, $k=1,2,\ldots$, $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$, $i=1,2,\ldots,I-1$, $j=1,2,\ldots,J$, $t=1,2,\ldots,T$.

Note that the larger number of binary variables, δ_{jihe}, the greater the computational load. It seems unnecessary to specify different promotion rate ranges for each age with the same total length of service and for each total length of service within the same grade. In practice, it is likely to be sufficient for total length of service within the same grade to be divided into, for instance, two with a separation point in terms of total length of service in grade i, H_{Si}, $H_{Pi}<H_{Si}<H_{Ui}$, decided by the decision makers to reflect practical considerations.

Let $B^{(k)}_{ji1}$ and $B^{(k)}_{j+1,i1}$ denote the lower bound and upper bound of jth possible range for promotion rate, p_{ihet}, at iteration k, where the total length of service h is in band 1, i.e. $H_{Pi}\le h\le H_{Si}$. Similarly, let $B^{(k)}_{ji2}$ and $B^{(k)}_{j+1,i2}$ denote the lower bound and upper bound of jth possible range for promotion rate, p_{ihet}, at iteration k, where the total length of service h is in band 2, i.e. $H_{Si}+1\le h\le H_{Ui}$. Let δ_{ji1} and δ_{ji2} denote the binary variables for jth possible ranges of promotion rate, p_{ihet}, in the band 1 and band 2 of total length of service respectively. Thus, if only two bands, defined in terms of total length of service, are considered, constraints (5-6a) to (5-6c) become:

$$-m_{ihet} + B^{(k)}_{ji1}n_{i,h-1,e-1,t-1} + M\delta_{ji1} \le M \tag{5-6a1}$$

$$m_{ihet} - B^{(k)}_{j+1,i1}n_{i,h-1,e-1,t-1} + M\delta_{ji1} \le M \tag{5-6b1}$$

$$\sum_{j=1}^{J} \delta_{ji1} = 1 \tag{5-6c1}$$

where $k=1,2,...,$ $h=H_{Pi},H_{Pi}+1,...,H_{Si}$, $e=A_L+h,A_L+h+1,...,\mathrm{Min}\{A_U+h,A_{Ri}\}$, $i=1,2,...,I-1$, $j=1,2,...,J$, $t=1,2,...,T$

$$-m_{ihet} + B^{(k)}_{ji2}n_{i,h-1,e-1,t-1} + M\delta_{ji2} \le M \tag{5-6a2}$$

$$m_{ihet} - B^{(k)}_{j+1,i2}n_{i,h-1,e-1,t-1} + M\delta_{ji2} \le M \tag{5-6b2}$$

$$\sum_{j=1}^{J} \delta_{ji2} = 1 \tag{5-6c2}$$

where $k=1,2,...,$ $h=H_{Si}+1,H_{Si}+2,...,H_{Ui}$, $e=A_L+h,A_L+h+1,...,\mathrm{Min}\{A_U+h,A_{Ri}\}$ $i=1,2,...,I-1$, $j=1,2,...,J$, $t=1,2,...,T$.

To simplify the model and reduce the computational effort, only one promotion rate range will be considered for each grade, i.e. independent of age and service length. Let $B_{ji}^{(k)}$ and $B_{j+1,i}^{(k)}$ denote the lower and upper bounds of the jth possible range for promotion rate, p_{ihet}, from grade i to i+1 at iteration k, and let δ_{ji} denote the binary variable for the jth possible promotion rate range from grade i to i+1 in any year, j=1,2,...,J, i=1,2,...,I-1. Thus, constraints (5-6a) to (5-6c) become:

$$-m_{ihet} + B_{ji}^{(k)} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \tag{5-6a3}$$

$$m_{ihet} - B_{j+1,i}^{(k)} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \tag{5-6b3}$$

$$\sum_{j=1}^{J} \delta_{ji} = 1 \tag{5-6c3}$$

where k=1,2,..., $h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui}$, $e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\}$ i=1,2,...,I-1, j=1,2,...,J, t=1,2,...,T.

5.3 THE FORM OF THE MIP MODEL

The entire MIP model for iteration k with a single promotion rate range for each grade, i.e. independent of age and service length, is summarised below.

Minimise

$$\sum_{t=1}^{T} C_{Rt}(1+\alpha)^{-t} r_t + \sum_{i=1}^{I} \sum_{t=1}^{T} \{ \sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h,A_{Ri}-1\}} C_{Siht}(1+\alpha)^{-t} n_{ihet} +$$

$$\sum_{h=H_{Li}+1}^{H_R-1} \sum_{e=A_L+h}^{A_U+h} C_{Fiht}(1+\alpha)^{-t}(W'_{ihet} n_{i,h-1,e-1,t-1}) +$$

$$\sum_{h=H_R}^{H_{Ui}} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h,A_{Ri}\}} \sum_{z=1}^{A_E-e} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} W'_{ihet} n_{i,h-1,e-1,t-1} +$$

$$\sum_{h=A_{Ri}-A_U}^{H_{Ui}} \sum_{e=A_{Ri}}^{A_{Ri}} \sum_{z=1}^{A_E-A_{Ri}} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} n_{ihet} \} \tag{5-7a}$$

subject to

$$n_{10et}-D_e r_t=0 \tag{5-7b1}$$

$e=A_L, A_L+1, \ldots, A_U,\ \forall t$

$$n_{1het}-(1-W_{1het})n_{1,h-1,e-1,t-1}=0 \tag{5-7b2}$$

$h=1,2,\ldots,H_{P1}-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,\ \forall t$

$$n_{1het}-(1-W_{1het})n_{1,h-1,e-1,t-1}+m_{1het}=0 \tag{5-7b3}$$

$h=H_{P1}, H_{P1}+1, \ldots, H_{U1},\ e=A_L+h, A_L+h+1, \ldots, \mathrm{Min}\{A_U+h, A_{R1}\},\ \forall t$

$$n_{Ihet}-(1-W_{Ihet})n_{I,h-1,e-1,t-1}-\theta_{I-1,h}m_{I-1,het}=0 \tag{5-7b4}$$

$h=H_{LI}+1, H_{LI}+2, \ldots, H_{UI},\ e=A_L+h, A_L+h+1, \ldots, \mathrm{Min}\{A_U+h, A_{RI}\},\ \theta_{I-1,h}=0 \text{ for } h>H_{UI-1},\ \theta_{I-1,h}=1 \text{ for } h\le H_{UI-1},\ \forall t$

$$n_{ihet}-\theta_{i-1,h}m_{i-1,het}=0 \tag{5-7b5}$$

$i=2,3,\ldots,I,\ h=H_{Li},\ e=A_L+h, A_L+h+1, \ldots, A_U+h,\ \theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-\theta_{i-1,h}m_{i-1,het}=0 \tag{5-7b6}$$

$i=2,3,\ldots,I-1,\ h=H_{Li}+1, H_{Li}+2, \ldots, H_{Pi}-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,$

$\theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$

$$n_{ihet}-(1-W_{ihet})n_{i,h-1,e-1,t-1}-\theta_{i-1,h}m_{i-1,het}+m_{ihet}=0 \tag{5-7b7}$$

$i=2,3,\ldots,I-1,\ h=H_{Pi}, H_{Pi}+1, \ldots, H_{Ui},\ e=A_L+h, A_L+h+1, \ldots, \mathrm{Min}\{A_U+h, A_{Ri}\},$

$\theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$

$$s_t - \sum_{i=1}^{I} \sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h, A_{Ri}-1\}} n_{ihet} = 0 \tag{5-7c1}$$

$$s_t \ge S_t(1-E_{Lt}) \qquad \forall t \tag{5-7c2}$$

$$s_t \le S_t(1+E_{Ut}) \qquad \forall t \tag{5-7c3}$$

$$\sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h, A_{Ri}-1\}} n_{ihet} \ge G_{it}S_t(1 - F_{Lit}) \qquad \forall i,t \tag{5-7d1}$$

$$\sum_{h=H_{Li}}^{H_{Ui}-1} \sum_{e=A_L+h}^{\mathrm{Min}\{A_U+h, A_{Ri}-1\}} n_{ihet} \le G_{it}S_t(1 + F_{Uit}) \qquad \forall i,t \tag{5-7d2}$$

$$r_t \ge R_{Lt} \qquad \forall t \tag{5-7e1}$$

$$r_t \le R_{Ut} \qquad \forall t \tag{5-7e2}$$

$$-m_{ihet} + B_{ji}^{(k)} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \tag{5-7f1}$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi}, H_{Pi}+1, \ldots, H_{Ui},\ e=A_L+h, A_L+h+1, \ldots, \mathrm{Min}\{A_U+h, A_{Ri}\},\ \forall t$

$$m_{ihet} - B^{(k)}_{j+1,i} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \qquad (5\text{-}7f2)$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ e=A_L+h,A_L+h+1,\ldots,\mathrm{Min}\{A_U+h,A_{Ri}\},\ \forall t$

$$\sum_{j=1}^{J} \delta_{ji} = 1 \qquad i=1,2,\ldots,I-1 \qquad (5\text{-}7f3)$$

$$n_{ihet},\ s_t,\ r_t,\ m_{ihet} \ge 0,\ \delta_{ji} = 0,1$$

The new form of the model (5-7), i.e. model (5-8), for use with XPRESS-MP is outlined in appendix B.

5.4 A MODEL INVOLVING GRADE AND SERVICE LENGTH

The model (5-7) involving grade, total length of service and age may consume a great deal of computational time and computer memory, and may not be easy to solve using a personal computer. In that case, a single recruitment age, with each total length of service associated with a single age, could be used to reduce the size of the model. This single age model can be obtained by simply specifying the same value for the minimum and maximum ages of recruitment in the model. However, for simplicity, age can be dropped from the model since it can be derived from recruitment age and total length of service. A model based on the model (5-7) and only involving grade and total length of service is outlined below.

Minimise

$$\sum_{t=1}^{T} C_{Rt}(1+\alpha)^{-t} r_t + \sum_{i=1}^{I} \sum_{t=1}^{T} \{ \sum_{h=H_{Li}}^{H_{Ui}-1} C_{Siht}(1+\alpha)^{-t} n_{iht} +$$

$$\sum_{h=H_{Li}+1}^{H_R-1} C_{Fiht}(1+\alpha)^{-t}(W'_{iht} n_{i,h-1,t-1}) +$$

$$\sum_{h=H_R}^{H_{Ui}} \sum_{z=1}^{A_E-h-A} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{h+A,h+A+z} W'_{iht} n_{i,h-1,t-1} +$$

$$\sum_{h=H_{Ui}}^{H_{Ui}} \sum_{e=A_{Ri}}^{A_{Ri}} \sum_{z=1}^{A_E-A_{Ri}} C_{Piht}(1+\beta)^z(1+\alpha)^{-(z+t)} Y_{e,e+z} n_{iht}\} \tag{5-9a}$$

subject to

$$n_{10t} - r_t = 0 \qquad \forall t \tag{5-9b1}$$

$$n_{1ht} - (1 - W_{1ht}) n_{1,h-1,t-1} = 0 \qquad h=1,2,\ldots,H_{P1}-1,\ \forall t \tag{5-9b2}$$

$$n_{1ht} - (1-W_{1ht}) n_{1,h-1,t-1} + m_{1ht} = 0 \tag{5-9b3}$$

$$h=H_{P1},H_{P1}+1,\ldots,H_{U1},\ \forall t$$

$$n_{Iht} - (1 - W_{Iht}) n_{I,h-1,t-1} - \theta_{I-1,h} m_{I-1,ht} = 0 \tag{5-9b4}$$

$$h=H_{LI}+1,H_{LI}+2,\ldots,H_{UI},\ \theta_{I-1,h}=0 \text{ for } h>H_{UI-1},\ \theta_{I-1,h}=1 \text{ for } h\le H_{UI-1},\ \forall t$$

$$n_{iht} - \theta_{i-1,h} m_{i-1,ht} = 0 \tag{5-9b5}$$

$$i=2,3,\ldots,I,\ h=H_{Li},\ \theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$$

$$n_{iht} - (1 - W_{iht}) n_{i,h-1,t-1} - \theta_{i-1,h} m_{i-1,ht} = 0 \tag{5-9b6}$$

$$i=2,3,\ldots,I-1,\ h=H_{Li}+1,H_{Li}+2,\ldots,H_{Pi}-1,\ \theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$$

$$n_{iht} - (1 - W_{iht}) n_{i,h-1,t-1} - \theta_{i-1,h} m_{i-1,ht} + m_{iht} = 0 \tag{5-9b7}$$

$$i=2,3,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ \theta_{i-1,h}=0 \text{ for } h>H_{Ui-1},\ \theta_{i-1,h}=1 \text{ for } h\le H_{Ui-1},\ \forall t$$

$$s_t - \sum_{i=1}^{I} \sum_{h=H_{Li}}^{H_{Ui}-1} n_{iht} = 0 \tag{5-9c1}$$

$$s_t \ge S_t(1 - E_{Lt}) \qquad \forall t \tag{5-9c2}$$

$$s_t \le S_t(1 + E_{Ut}) \qquad \forall t \tag{5-9c3}$$

$$\sum_{h=H_{Li}}^{H_{Ui}-1} n_{iht} \ge G_{it} S_t (1 - F_{Lit}) \qquad \forall i,t \tag{5-9d1}$$

$$\sum_{h=H_{Li}}^{H_{Ui}-1} n_{iht} \le G_{it} S_t (1 + F_{Uit}) \qquad \forall i,t \tag{5-9d2}$$

$$r_t \ge R_{Lt} \qquad \forall t \tag{5-9e1}$$

$$r_t \le R_{Ut} \qquad \forall t \tag{5-9e2}$$

$$-m_{iht} + B_{ji}^{(k)} n_{i,h-1,t-1} + M\delta_{ji} \le M \tag{5-9f1}$$

$$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ \forall t$$

$$m_{iht} - B_{j+1,i}^{(k)} n_{i,h-1,t-1} + M\delta_{ji} \le M \tag{5-9f2}$$

$$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,H_{Ui},\ \forall t$$

$$\sum_{j=1}^{J} \delta_{ji} = 1 \qquad i=1,2,\dots,I-1 \tag{5-9f3}$$

$$n_{iht},\ s_t,\ r_t,\ m_{iht} \geq 0$$

$$\delta_{ji} = 0,1$$

In model (5-9), the definitions of parameters and variables are similar to that in sections 5.2.2 and 5.2.3 except for the dropping of the age index, e. The single recruitment age, defined as A, in the objective function of the model (5-9a) could be the mode or average of the recruitment ages. The costs of the annual pension in year t include the payments in years until death after leaving the system. Therefore, it is necessary to know the maximum survival time of the individuals after leaving the system. This maximum survival time can be obtained by subtracting the age of the individuals leaving the system, i.e. h+A, from the maximum life expectancy, i.e. A_E. Note that the maximum total length of service in grade i at retirement, defined as H_{Ui}, is given by subtracting the single recruitment age from the retirement age, i.e. $H_{Ui}=A_{Ri}-A$, i=1,2,...,I.

5.5 SUMMARY

Different promotion and retirement policies in terms of total length of service and retirement ages have cost implications and also cause problems in the demand for, and supply of, manpower, and affect the career prospects of individuals. In some systems, such as the police and the armed forces, total length of service is an important qualification for promotion. In this chapter, a model based on the MIP model described in Chapter 3 and involving age and total length of service in each grade has been developed to determine the minimum cost recruitment and promotion policies that will satisfy the manpower requirements while ensuring promotion rates are approximately constant from year to year. The requirement for stable promotion rates over time can be modelled by imposing constraints for each grade, total length of service, age and year to ensure that the promotion rate for that grade, total length of service and age is in the same range in each year. To simplify the model and reduce computational effort, the same promotion rate range was specified for each total length of service and age in the same grade. However, total length of service within the same grade can be divided into a number of promotion bands, the boundaries of the bands being specified by the decision makers in terms of length of service and age. A simplified model involving a single promotion band for each total length of service and age in the same grade has also been presented. If the size of this model is too large to be solved on a personal computer, a single recruitment age can be considered so that each total length of service is associated with a single age. This single age could, for example, be the mode of the recruitment ages.

Manpower planning should not only consider organisational goals but should also take account of the career development of the individuals within the organisation. In particular, stable promotion rates over time help demonstrate to individuals that their career prospects do not depend on factors which are outside their control, e.g. the time at which they entered the organisation. Career development opportunities can be considered in terms of the probabilities of eventual promotion to a specified grade, and the expected waiting time to reach that grade. Although the MIP model ensures that promotion rates are as stable as possible over time, it gives no direct information on the probability and expected waiting time for eventual promotion. Methods for obtaining the probability and expected waiting time for promotion will be developed in Chapter 6. A decision support system for generating and evaluating manpower policies, and producing the probability and expected waiting time will also be considered in Chapter 6.

CHAPTER 6

TOWARDS A DECISION SUPPORT SYSTEM FOR MANPOWER PLANNING

6.1 INTRODUCTION

A model involving age and total length of service in each grade has been developed to determine the minimum cost recruitment, retirement and promotion policies that will satisfy the manpower needs while ensuring that promotion rates remain stable over time. The results from this model give no direct information on the likelihood that any given individual will ultimately be promoted. However, the probability of promotion and the expected waiting time before promotion can be derived from the model results. Methods for obtaining the probability and expected waiting time before eventual promotion are now developed.

Note that if the probability of eventual promotion to a specified grade or the expected waiting time to reach that grade is unacceptable to the decision makers, alternative manpower policies should be developed and the model re-run until it is satisfactory to the decision makers. Different manpower policies, defined in terms of factors such as required minimum total length of service for promotion, retirement age, and target proportion of staff in each grade, will result in different manpower availabilities and career development opportunities for the people within the organisation. In practice it may be desirable to have a range of acceptable manpower policies, as this may offer management the opportunity to choose the policies that best suit their situation. A personal computer based decision support system for evaluating manpower policies and calculating the probability of promotion and the associated expected waiting time is now developed.

6.2 THE PROBABILITY AND EXPECTED WAITING TIME FOR PROMOTION

The promotion rate gives information on the chance of being promoted at a particular time but gives little direct information on career prospects. It is useful for individuals and management to have information on both the probability of promotion and the expected waiting time before ultimate promotion. This information will help management to review manpower policies and assist individuals in making decisions on whether to remain in the organisation. The probability of eventual promotion will also give information on the expected proportion of entrants to the grade who will leave that grade by promotion.

For simplicity, a single recruitment age, A, will be considered in the following sections. This age could be the mode or average of the recruitment ages. The maximum total length of service of staff in grade i at retirement, H_{Ui}, is obtained by subtracting the single recruitment age from the retirement age, A_{Ri}, i.e. $H_{Ui}=A_{Ri}-A$, $i=1,2,\ldots,I$. It is assumed that the maximum and minimum total length of service of staff in grade i+1 are greater than that in the grade i, i.e. $H_{U,i+1}>H_{Ui}$, and $H_{L,i+1}>H_{Li}$, $i=1,2,\ldots,I-1$. It is also assumed that promotions only occur to the next higher grade and that demotions are not allowed.

6.2.1 The Probability of Being in a Specified Grade

It is assumed that the transition, or promotion, process of individuals is a Markov chain, i.e. the probability of transition from one grade to another is dependent only on the current grade, and that the transition probabilities, or promotion probabilities, are independent of time.

Let $\{X_t, t=0,1,2,...\}$ be a transition process indexed by time t, where X_t is the state of an individual at the end of year t, i.e. the grade associated with total length of service in which the individual will be at the end of year t. Let $\mathbf{P\{X_t=j\cdot h+1|X_{t-1}=i\cdot h\}}$ denote the probability that at the end of year t an individual will be in grade j with total length of service h+1, given that at the end of year t-1 the individual is in grade i with total length of service h, where $i=1,2,...,I$, $i\le j\le I$, $t=1,2,...$, $H_{Li}\le h\le H_{Ui}-1$, and H_{Li} and H_{Ui} are the minimum and maximum total length of service of staff in grade i respectively. This probability is often termed the one step transition probability or the probability of making the transition from $i\cdot h$ to $j\cdot h+1$ in one time period, i.e. year. According to the Markovian property, it has

$$P\{X_t=i_t\cdot h+t|X_{t-1}=i_{t-1}\cdot h+t-1,X_{t-2}=i_{t-2}\cdot h+t-2,...,X_0=i_0\cdot h\}$$
$$= P\{X_t=i_t\cdot h+t|X_{t-1}=i_{t-1}\cdot h+t-1\}$$

for all t, where $1\le i_0\le i_1\le ...\le i_t\le I$. In other words, the conditional probability of X_t given the whole past history of the transition process must be equal to the conditional probability of X_t given X_{t-1}, i.e. the information of states at the end of year t-1 is sufficient to predict the states at the end of year t. Since the transition probabilities are assumed to be stationary, i.e. they do not change with the passage of time, the probability $P\{X_t=j\cdot h+1|X_{t-1}=i\cdot h\}$ will be written as below for convenience:

$$P_{i\cdot h,j\cdot h+1}=P\{X_t=j\cdot h+1|X_{t-1}=i\cdot h\}$$

This notation is similar to Phillips, Ravindran and Solberg (1976).

Note that for an individual in grade i at the end of year t-1 with total length of service h, then at the end of year t the individual can be promoted to grade i+1, or can leave the system, or can remain in the same grade, i.e. grade i, and the total length of service in each case will increase by one year, i.e. to h+1. This probability of promotion to grade i+1, i.e. $P_{i\cdot h,i+1\cdot h+1}$, can be estimated by averaging the promotion rates generated by the model (5-9) during T planning years, i.e.

$$P_{i\cdot h,i+1\cdot h+1}=\sum_{t=1}^{T} m_{i,h+1,t}/\sum_{t=1}^{T} n_{ih,t-1} \tag{6-1a}$$

$$i=1,2,...,I-1,\ H_{Li}\le h\le H_{Ui}-1$$

where $m_{i,h+1,t}$ is the number of staff promoted from grade i to i+1 with total length of service h+1 at the end of year t; $n_{ih,t-1}$ is the number of staff in grade i with total length of service h at the end of year t-1; H_{Li} and H_{Ui} are the minimum and maximum total length of service of staff in grade i respectively; i.e. in the planning horizon of T years, the total number of staff promoted from a specified grade divided by the sum of staff in post in that grade at the start of each year of this planning horizon is used as an estimator of the average promotion rate. Note that the promotion rate is a ratio, and that other methods, in particular the geometric mean, are often used in averaging ratios (e.g. Parson, 1974).

Let $W_{i\cdot h}$ denote the wastage rate of staff in grade i with total length of service h in any year, $i=1,2,...,I$, $H_{Li}+1\le h\le H_{Ui}$. This wastage rate is estimated from historic data. The probability that at the end of year t an individual will remain in the same grade with total length of service h+1, given that at the end of year t-1 the individual is in grade i with total length of service h, i.e. $P_{i\cdot h,i\cdot h+1}$, can be obtained then as below:

$$P_{i\cdot h,i\cdot h+1}=1-P_{i\cdot h,i+1\cdot h+1}-W_{i\cdot h+1} \qquad i=1,2,\ldots,I-1,\ H_{Li}\leq h\leq H_{Ui}-1 \qquad (6\text{-}1b)$$

$$P_{I\cdot h,I\cdot h+1}=1-W_{I\cdot h+1} \qquad H_{LI}\leq h\leq H_{UI}-1 \qquad (6\text{-}1c)$$

Note that in equation (6-1a), when total length of service h+1 in grade i+1 is smaller than the minimum total length of service in this grade, the probability $P_{i\cdot h,i+1\cdot h+1}$ will be zero, i.e.

$$P_{i\cdot h,i+1\cdot h+1}=0 \qquad h<H_{L,i+1}-1 \qquad (6\text{-}1d)$$

Similarly,

$$P_{i\cdot h,i\cdot k}=0 \qquad k\neq h+1 \qquad (6\text{-}1e)$$

$$P_{i\cdot h,i+1\cdot k}=0 \qquad k\neq h+1 \qquad (6\text{-}1f)$$

$$P_{i\cdot h,j\cdot k}=0 \qquad j\neq i,i+1 \qquad (6\text{-}1g)$$

The transition probabilities matrix then can be constructed from equations (6-1).

The probability $P\{X_t=j\cdot h+1|X_{t-1}=i\cdot h\}=P_{i\cdot h,j\cdot h+1}$ is referred to as the one step transition probability, since it describes the system between time t-1 and t. A t-step transition probability is then defined by

$$P\{X_t=j\cdot h+t|X_0=i\cdot h\}=P^{(t)}_{i\cdot h,j\cdot h+t}$$

Obviously, the total length of service of staff will increase by t years after a t-step transition. Thus,

$$P^{(t)}_{i\cdot h,j\cdot k}=0 \qquad k\neq h+t \qquad (6\text{-}2a)$$

It has been noted that after a one step transition, the total length of service of staff will increase by one year, and the individual can be promoted to the next higher grade, or can leave the system, or can remain in the same grade. Clearly, the individual will be in grade j with total length of service h+t at the end of year t only in the cases when the individual is in grade j-1 or in grade j with total length of service h+t-1 at the end of year t-1. The probability that at the end of year t an individual will be in grade j with total length of service h+t, given that at the current time (year 0) the individual is in grade i with total length of service h, i.e. $P\{X_t=j\cdot h+t|X_0=i\cdot h\}$, can be derived as below:

$$P^{(1)}_{i\cdot h,i\cdot h+1} = P_{i\cdot h,i\cdot h+1} \qquad i=1,2,\ldots,I,\ H_{Li}\le h\le H_{Ui}-1 \qquad (6\text{-}2b)$$

$$P^{(1)}_{i\cdot h,i+1\cdot h+1} = P_{i\cdot h,i+1\cdot h+1} \qquad i=1,2,\ldots,I-1,\ H_{Li}\le h\le H_{Ui}-1 \qquad (6\text{-}2c)$$

$$\begin{aligned}
P^{(t)}_{i\cdot h,j\cdot h+t}&=P\{X_t=j\cdot h+t|X_0=i\cdot h\}\\
&= P\{X_{t-1}=j-1\cdot h+t-1,X_t=j\cdot h+t|X_0=i\cdot h\}+P\{X_{t-1}=j\cdot h+t-1,X_t=j\cdot h+t|X_0=i\cdot h\}\\
&= P\{X_{t-1}=j-1\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j-1\cdot h+t-1,X_0=i\cdot h\}+\\
&\quad P\{X_{t-1}=j\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j\cdot h+t-1,X_0=i\cdot h\}\\
&= P\{X_{t-1}=j-1\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j-1\cdot h+t-1\}+\\
&\quad P\{X_{t-1}=j\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j\cdot h+t-1\}\\
&= P^{(t-1)}_{i\cdot h,j-1\cdot h+t-1}P_{j-1\cdot h+t-1,j\cdot h+t}+P^{(t-1)}_{i\cdot h,j\cdot h+t-1}P_{j\cdot h+t-1,j\cdot h+t} \qquad (6\text{-}2d)
\end{aligned}$$

where $i=1,2,\ldots,I-1$, $i+1\le j\le I$, $H_{Li}\le h\le H_{Ui}-1$, $\mathrm{Max}\{j-i,\ 2,\ H_{Lj}-h\}\le t\le H_{Uj}-h$; $P_{j-1\cdot h+t-1,j\cdot h+t}=0$ for $t\ge H_{U,j-1}-h+1$, and $P_{j\cdot h+t-1,j\cdot h+t}=0$ for $t\le H_{Lj}-h$. Similarly,

$$\begin{aligned}
P^{(t)}_{i\cdot h,i\cdot h+t}&= P\{X_t=i\cdot h+t|X_0=i\cdot h\}\\
&= P\{X_{t-1}=i\cdot h+t-1,X_t=i\cdot h+t|X_0=i\cdot h\}\\
&= P\{X_{t-1}=i\cdot h+t-1|X_0=i\cdot h\}P\{X_t=i\cdot h+t|X_{t-1}=i\cdot h+t-1,X_0=i\cdot h\}\\
&= P\{X_{t-1}=i\cdot h+t-1|X_0=i\cdot h\}P\{X_t=i\cdot h+t|X_{t-1}=i\cdot h+t-1\}\\
&= P^{(t-1)}_{i\cdot h,i\cdot h+t-1}P_{i\cdot h+t-1,i\cdot h+t} \qquad (6\text{-}2e)
\end{aligned}$$

where $i=1,2,\ldots,I$, $H_{Li}\le h\le H_{Ui}-1$, $2\le t\le H_{Ui}-h$. The probabilities can therefore be calculated recursively using estimates of one step probabilities derived from model output using (6-1a).

Note that in equation (6-2d), the total length of service of the individual in grade j, i.e. h+t, is within his or her minimum total length of service, H_{Lj}, and maximum total length of service, H_{Uj}, i.e.

$$H_{Lj} \leq h+t \leq H_{Uj}$$

Since promotion only occurs to the next higher grade at the end of each year, the minimum number of years for promotion from grade i to grade j is j-i.

6.2.2 The Probability of Eventual Promotion to a Specified Grade

The t-step transition probabilities give information on the chance of individuals being in a specified grade after t years. Information of more interest to staff is the probability that an individual is ultimately promoted to a specified higher grade, given that the individual is at particular grade at a given time. For example, this specified higher grade could be the grade that the individual aspires to reach. This information will help individuals to evaluate their career prospects and to decide whether they should remain in the system or not.

Let $\mathbf{P_t\{j|X_0=i \cdot h\}}$ denote the probability that an individual will be promoted to grade j at the end of year t, given that at the current time (year 0) the individual is in grade i with total length of service h, where $i=1,2,\ldots,I-1$, $i+1 \leq j \leq I$, $H_{Li} \leq h \leq H_{Ui}-1$, $\mathrm{Max}\{j-i, H_{Lj}-h\} \leq t \leq H_{U,j-1}-h$. At the end of year 0 the individual has total length of service h. Clearly, at the end of year t the individual will have total length of service h+t. The individual can be promoted to grade j at the end of year t only when the individual is in grade j-1 with total length of service h+t-1 at the end of year t-1. Thus,

$$P_1\{i+1|X_0=i.h\} = P_{i \cdot h, i+1 \cdot h+1} \qquad i=1,2,\ldots,I-1,\ H_{Li} \leq h \leq H_{Ui}-1 \qquad (6\text{-}3a)$$

$$P_t\{j|X_0=i\cdot h\} = P\{X_{t-1}=j-1\cdot h+t-1, X_t=j\cdot h+t|X_0=i\cdot h\}$$
$$= P\{X_{t-1}=j-1\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j-1\cdot h+t-1, X_0=i\cdot h\}$$
$$= P\{X_{t-1}=j-1\cdot h+t-1|X_0=i\cdot h\}P\{X_t=j\cdot h+t|X_{t-1}=j-1\cdot h+t-1\}$$
$$= P^{(t-1)}_{i\cdot h, j-1\cdot h+t-1}P_{j-1\cdot h+t-1, j\cdot h+t} \qquad (6\text{-}3b)$$

where $i=1,2,\ldots,I-1$, $i+1\le j\le I$, $H_{Li}\le h\le H_{Ui}-1$, $Max\{j-i,2,H_{Lj}-h\}\le t\le H_{U,j-1}-h$.

Let **$\underline{P}(j|X_0=i\cdot h)$** denote the probability of an individual ultimately being promoted to grade j, given that at current time (year 0) the individual is in grade i with total length of service h, where $i=1,2,\ldots,I-1$, $i+1\le j\le I$, $H_{Li}\le h\le H_{Ui}-1$. The probability $\underline{P}(j|X_0=i\cdot h)$ is the summation over all years t of the probabilities $P_t(j|X_0=i\cdot h)$, that is

$$\underline{P}(j|X_0=i\cdot h) = \sum_{t=k}^{H_{U,j-1}-h} P_t(j|X_0=i\cdot h) \qquad (6\text{-}4)$$

where $k=Max\{j-i,H_{Lj}-h\}$, $i=1,2,\ldots,I-1$, $i+1\le j\le I$, $H_{Li}\le h\le H_{Ui}-1$.

6.2.3 The Expected Waiting Time for Promotion

Having considered the probability of eventual promotion, individuals may be interested to know how long it is likely to take for promotion to occur, i.e. the expected waiting time before promotion for those who will be promoted. This can be derived from the probability $\underline{P}(j|X_0=i\cdot h)$.

Given that at the current time (year 0) an individual is in grade i with total length of service h, and also given that the individual will be promoted to grade j ultimately, the probability of the individual being promoted to grade j at the end of year t is:

$$P_t(j|X_0=i\cdot h)/\underline{P}(j|X_0=i\cdot h) \qquad (6\text{-}5)$$

$i=1,2,\ldots,I-1$, $i+1\le j\le I$, $H_{Li}\le h\le H_{Ui}-1$, $Max\{j-i,H_{Lj}-h\}\le t\le H_{U,j-1}-h$

Let $\mathbf{E_w(j|X_0=i \cdot h)}$ denote the expected waiting time of an individual who will be promoted to grade j, given that at the current time (year 0) the individual is in grade i with total length of service h, $i=1,2,\ldots,I-1$, $i+1 \le j \le I$, $H_{Li} \le h \le H_{Ui}-1$. Thus,

$$E_w(j|X_0=i \cdot h) = \sum_{t=k}^{H_{U,j-1}-h} tP_t(j|X_0=i \cdot h)/\underline{P}(j|X_0=i \cdot h) \qquad (6\text{-}6)$$

where $k=\mathrm{Max}\{j-i, H_{Lj}-h\}$, $i=1,2,\ldots,I-1$, $i+1 \le j \le I$, $H_{Li} \le h \le H_{Ui}-1$.

Note that as with the probabilities of promotion derived in section 6.2.2, the expected waiting time before promotion derived above can be calculated from the output of the MIP model developed in Chapter 5.

6.3 A DECISION SUPPORT SYSTEM FOR MANPOWER PLANNING

A personal computer based decision support system for manpower planning has been developed based on the MIP model described in Chapter 5 with output presented in the form of graphs and tables to provide information to allow management to evaluate manpower policies.

6.3.1 The Modules of the Decision Support System

The decision support system consists of seven major modules, namely,

1. model builder
2. model optimiser
3. results writer
4. graph generator
5. table generator
6. bounds generator

7. overlapping ranges generator

The MIP models (3-18) and (5-9) are set up (appendices C1 and C2) and solved by using the model builder and optimiser of XPRESS-MP (Dash Associates, 1991). The model builder allows the model to be specified in a form similar to the mathematical statement of the model, and creates a matrix file for input to the optimiser. The optimiser seeks an optimal solution for the model and writes the results to ASCII files. Since the results in this ASCII file are unreadable, the results writer, a computer program in BASIC (appendix C3), is used to write the results in the form for graph and table generator to different files. The results writer is also used to generate a file for calculating the probability and expected waiting time before promotion (appendix C4) for use in evaluating the career prospects of individuals.

The graph generator (appendix C3) produces output for input to Harvard Graphics (e.g. Campbell, 1990), a PC based graphical software package, to produce a number of graph types with features such as titles, legends and footnotes. These graphs include:

1. number of staff in each grade in each year
2. number of promotions in each grade in each year
3. total staff in each year
4. number of recruits in each year
5. manpower costs in each year
6. promotion rate in each grade and year

The table generator can be any word processor, for instance, WordStar. The results in the form of table can be read by using the word processor. These tables include:

1. number of staff in each grade, for each total length of service and

year

2. number of promotions in each grade, for each total length of service and year
3. number leaving each grade, for each total length of service and year
4. number retiring from each grade in each year
5. promotion rate in each grade for each total length of service and year
6. average promotion rate over T planning years in each grade, and for each total length of service
7. the probability of eventual promotion
8. the expected waiting time before eventual promotion

In the iterative procedure for solving the MIP model described in section 3.5, the lower and upper bounds of the promotion rate ranges must be specified and the range width must be reduced at successive iterations. The bounds generator, a computer program in BASIC (appendix C3), is used to generate the lower and upper bounds at each iteration, and then automatically incorporate these bounds into the model builder. The procedure described in section 3.5 is used to find an acceptable solution, i.e. an optimal solution in which the promotion rate range width is acceptable to the decision makers, or until the procedure stops because no feasible solution of an acceptable range width can be found. As has been noted, even when an acceptable solution is found, it may be possible to find a lower cost solution, and in cases where no acceptable feasible solution is found, it may be possible to find a feasible solution by specifying other sets of promotion rate ranges with an acceptable range width. The overlapping ranges generator (appendix C3) is used to generate alternative bounds for promotion rates by using the method described in section 4.3, and then automatically incorporate these bounds into the model builder.

6.3.2 Use of the Decision Support System

The procedure for using the decision support system is presented in figure 6.1. The model builder is first invoked to set up the MIP model and then the model is solved by calling the optimiser. If a solution is found and the promotion rate range width and the manpower cost are satisfactory to the decision makers, the results writer is invoked to create files for use by the graph generator and the table generator, and to input to the program for calculating the probability and expected waiting time for promotion. The graph generator and table generator are then invoked to plot and tabulate the results. In cases where the results are not acceptable, e.g. the expected waiting time for promotion is too large, the development of alternative manpower policies should be considered. In these cases, the parameters of the MIP model must be revised, i.e. different sets of the parameters representing different manpower policies, the model builder re-invoked and the process repeated; otherwise, stop the process and leave the system.

In cases where a solution has been found and the promotion rate range width is acceptable to the decision makers, but a lower cost solution with the same range width is desirable, the overlapping range generator is invoked and the model builder is then called to repeat the process. In cases where a solution has been found but the promotion rate range width is not acceptable to the decision makers, the bounds generator is invoked to reduce the width and generate alternative bounds for promotion rate, and then the model builder is again used and the process repeated.

In cases where no feasible solution can be found but it is desired to search for a feasible solution with the same range width, either the overlapping ranges generator should be called or the parameters of the MIP model should be revised. The process is then continued by using the model builder.

The commands used in the decision support system for the invocation of the model builder, optimiser, bounds generator, overlapping ranges generator, results writer, graph generator and table generator for using the case of solving the MIP model (5-9) are prescribed in appendix D.

6.4 SUMMARY

The promotion rate gives information on the chance of individuals being promoted at a particular time but gives little direct information on career development of individuals. In this chapter, methods for obtaining the probability of individuals ultimately being promoted and the expected waiting time before promotion have been developed in order to evaluate the career prospects of individuals. The information on the probability and expected waiting time before eventual promotion will help management to evaluate the manpower policies and help individuals to make decisions on whether to remain in the organisation. In the case where the probability and expected waiting time are unacceptable to the decision makers or individuals, alternative manpower policies, e.g. involving reducing the required minimum total length of service for promotion, should be developed, and the model re-run until the results are satisfactory to the decision makers.

A personal computer based decision support system for evaluating manpower policies and calculating the probability and expected waiting time before eventual promotion has been developed. The results from this system can be presented in the form of tables and graphs. A case study of using the decision support system for military manpower planning will be illustrated in Chapter 7.

CHAPTER 7

A CASE STUDY OF MANPOWER PLANNING IN A MILITARY SYSTEM

7.1 INTRODUCTION

The MIP based manpower planning model described in Chapter 3, and the extended version developed in Chapter 5 to include age and total length of service in each grade, can be used to determine the minimum cost recruitment and promotion policies that will satisfy the manpower needs while ensuring that promotion rates remain stable over time. The use of the model is now demonstrated based on a case study of the officers in a military manpower system, although for reasons of confidentiality the data have been modified. In this case study, the effects on both costs and the career development opportunities of individuals of changes in the policy on the minimum total length of service required for pension entitlement are also considered.

7.2 SYSTEM DESCRIPTION

The military system is similar to that used to illustrate the use of the model developed in chapter 3. Only officer ranks are considered. These ranks are classified as second lieutenant, subaltern, captain, major, lieutenant colonel, and colonel or higher ranks. The ranks of colonel and above are lumped together because of the small numbers involved. For convenience these six ranks will be referred to as grades 1 to 6, with grade 1 denoting the lowest officer grade (i.e. second lieutenant) and grade 6 denoting the highest grade (i.e. colonel and above).

In the system, recruitment occurs only into grade 1 and the recruits must be trained for four years before they become fully operational. Only fully trained officers are considered in the model, so that the recruitment in year t in the model represents recruitment which actually occurred in year t-4, i.e. four years earlier. The variation in recruitment ages is slight and the recruitment age distribution is stable over time. Therefore, a single recruitment age is considered so that each total length of service is associated with a single age.

Promotion from grade i, $i=1,2,\ldots,5$, is made only into the grade immediately above, i.e. grade i+1. Staff can be considered for a promotion only after completing the minimum total length of service required for that promotion. Demotions from any grade are not allowed. Since staff morale is likely to be affected if promotion opportunities vary significantly from one year to another, it is desirable that promotion rates are stable over time. Wastage and retirement occur in all grades. Staff wastage results from ill-health, death and those who leave the system of their own choice. The highest grade, i.e. grade 6, loses staff only by wastage and retirement. Retirement age is a non-decreasing function of grade. Each individual must sign a ten year service contract on joining the military system. This contract may be renewed on termination, subject to the manpower requirements.

The costs in the system consist of recruitment costs, salary costs and pension costs. The recruitment cost includes the costs incurred in the recruitment process and the cost of training recruits before they become fully operational. The recruitment cost is a function of the number of recruits. The salary cost of all staff in the system, is a stock cost, the salary of an individual depending on grade and length of service. The pension cost involves the cost of lump sum payments and the cost of annual pensions. Staff who leave with a total length of service which is less than the minimum total length of service required for annual pension entitlement, i.e. 20 years, receive a single lump sum payment, this sum depending on salary on leaving the system. Pension payments, which are a proportion of salary on leaving the system, are made annually and adjusted in accordance with inflation until death. The dependent relatives of those who die in service or after retirement do not receive either a lump sum payment or an annual pension.

The military system is concerned with determining minimum cost recruitment, promotion and pension strategies which satisfy manpower requirements, while ensuring that promotion rates remain stable over time and that the career development opportunities for individuals remain at an acceptable level.

7.3 DATA FOR THE MANPOWER PLANNING MODEL

The number of staff initially in grade i, $i=1,2,\ldots,6$, with total length of service h, i.e. N_{ih0}, is presented in table 7.1. Note that in table 7.1, the number of staff in grade 1 with total length of service of 1 year and more have been lumped together because of the small numbers involved. Similarly in grades 2, 3, 4, 5 and 6 staff with total length of service of 6, 12, 20, 24 and 27 years and more respectively have been lumped together. The modal recruitment age, A, the target total number of staff, S_t, the maximum upper and lower proportional deviations in target total number of staff, E_{Ut} and E_{Lt}, the maximum upper and lower proportional deviations in target number of staff in grade i, F_{Uit} and F_{Lit}, the upper and lower bounds on the number of recruits to grade 1, R_{Ut} and R_{Lt}, the target proportion of staff in grade i, G_{it}, and the required minimum total length of service for annual pension entitlement, H_R, maintain the same values in each year t. The values of these parameters are given below:

$A=23$, $S_t=30000$, $E_{Ut}=0.15$,

$E_{Lt}=0.15$, $F_{Uit}=0.15$, $F_{Lit}=0.15$,

$R_{Ut}=3000$, $R_{Lt}=2000$, $G_{1t}=0.09$,

$G_{2t}=0.27$, $G_{3t}=0.25$, $G_{4t}=0.20$,

$G_{5t}=0.13$, $G_{6t}=0.06$, $H_R=20$

where $i=1,2,...,6$, $t=1,2,...,T$.

The minimum total length of service required for promotion from grade i, H_{Pi}, the minimum total length of service of staff in grade i, H_{Li}, the maximum total length of service in grade i at retirement, H_{Ui}, where $H_{Ui}=A_{Ri}-A$, and the retirement age, A_{Ri}, are presented in table 7.2. The wastage rate in grade i of staff with total length of service h at the end of year t, W_{iht}, is presented in table 7.3 and assumed to be the same in each year. In table 7.3, the increased wastage rate at total length of service 10 years due to contract termination can be seen. Note also that the wastage rates for total length of service approaching 20 years are small, and that these wastage rates increase dramatically on reaching 20 years. This is because staff must complete 20 years of service to obtain an annual pension, and therefore there is an additional incentive to stay in the system until total length of service is 20 years.

The average recruitment cost per person in year t, C_{Rt}, the average annual salary per person in grade i with total length of service h in year t, C_{Siht}, the average lump sum payment and annual pension per person for those who leave the system in grade i with total length of service h in year t, C_{Fiht}, and C_{Piht} respectively, are given by:

$$C_{Rt}=C_{R0}(1+\beta)^t$$

$$C_{Siht}=(\Gamma_i C_{SR}+hC_{SH})(1+\beta)^t$$

$$C_{Fiht}=0.4hC_{Siht}$$

$$C_{Piht}=0.8C_{Siht}$$

where $i=1,2,\ldots,6$, $t=1,2,\ldots T$, $h=H_{Li},H_{Li+1},\ldots,H_{Ui}$. The average recruitment cost per person in year 0, C_{R0}, the salary rate in grade i as a multiple, Γ_i, of the salary, C_{SR}, for recruits, the annual salary increment per year of service, C_{SH}, the annual rate of salary increase, β, the discount rate, α, and the maximum life expectancy, A_E, are given below:

$C_{R0}=20$, $C_{SR}=10$, $C_{SH}=0.5$,

$\Gamma_1=1$, $\Gamma_2=1.1$, $\Gamma_3=1.3$

$\Gamma_4=1.5$, $\Gamma_5=1.8$, $\Gamma_6=2$,

$\beta=0.06$, $\alpha=0.05$, $A_E=85$.

The probabilities of survival from age e to age e+1, i.e. $Y_{e,e+1}$, $e=23,24,\ldots,85$, are presented in table 7.4. These probabilities are derived from national male mortality statistics.

7.4 SOLUTION OF THE MODEL

The model was set up and solved using the decision support system described in Chapter 6. This system is composed of model builder, model optimiser, bounds generator, overlapping ranges generator, results writer, graph generator and table generator. To ensure that the career prospects of the individuals within the military system do not depend on the time at which they enter the system, constraints were imposed to ensure that the promotion rate in grade i, i=1,2,...,5, for any total length of service is in the same range in each year. At the first iteration, two possible promotion rate ranges of width 0.5 were considered. The optimal ranges at each iteration are presented in table 7.5. The solution procedure was stopped at iteration 6 when the range width, i.e. 0.0156, was regarded as satisfactory.

Case 1

For case 1, where at least 20 years total length of service is required for pension entitlement, the results in terms of the number of staff, the number of promotions, number of recruits, manpower cost, and promotion rates for the 10 year planning horizon are presented in figures 7.1 to 7.7. The detailed results in each grade with total length of service are presented in tables 7.6 to 7.11. For simplicity, only the number of staff at end of year 9 and detailed results at the end of year 10 are shown. The number of retirements in each grade in each year and the manpower cost in each year are presented in tables 7.12 and 7.13 respectively. The promotion probabilities and expected waiting time before eventual promotion to a specified grade are presented in tables 7.14 to 7.20.

In figure 7.3 it can be seen that the total number of staff in each year decrease gradually by approximately 350 staff on average per year. This gradual contraction in the size of the system is more practical than a sudden dramatic change. It can be seen from figure 7.1 that the number of staff in grade 3 and 5 decreases step by step and eventually achieves a stable level. The number of staff in grade 2 is reasonably stable after year 2. The numbers of staff in grades 4 and 6 increase during the first five years and then decrease. Even though the distribution of staff in each grade is different in each year, as shown in figure 7.7, the promotion rates remain stable over the planning horizon. Detailed promotion rates for each total length of service at the end of year 10 are presented in table 7.9. Note that in table 7.9, the promotion rate of staff promoted from grade 4 at a total length of service of 21 years is 0.2857, which exceeds the range width, 0.2188 to 0.2344, presented in table 7.5. This is because of the small number of staff involved and rounding error. Note also that in table 7.9, the average promotion rate in grade i at the end of year 10 is obtained from the total number of staff promoted from grade i at end of year 10 divided by those who are entitled to promotion in terms of total length of service in grade i at the end of year 9. The average promotion rates over 10 years in each grade for each total length of service, are presented in table 7.10.

In table 7.14 it can be seen that 99.19% of recruits can expect to be promoted to grade 2 and that 7.61% of them will subsequently be promoted to grade 6. From table 7.15, the expected waiting time for promotion to grade 2 is 1.06 years and to grade 6 is 17.37 years, i.e. 3.37 years more than the minimum total length of service required for promotion to grade 6. Other information which may help individuals in deciding whether to join or remain in the system can be obtained from tables 7.14 to 7.20.

Case 2

It has been noted that the pension cost is composed of the discounted cost of the lump sum payments and annual pensions. Since the annual pension is paid annually until death, it is a long-term financial burden on the system. As can be seen in table 7.13, the stock cost, recruitment cost, lump sum payment, and annual pension cost are 44.43%, 4.65%, 13.99% and 36.93% respectively of the total cost over 10 years. In figure 7.6, it can be seen that the annual pension as a proportion of the total cost increases after year 2 and then decreases slightly after year 8. To reduce the financial pressure, a policy of extending minimum total length of service required for pension entitlement from 20 to 22 years is considered in case 2. Because of the impact of the minimum total length of service required for pension entitlement on wastage rate, the actual wastage rates estimated for 20 years total length of service were used as the wastage rates for 22 years length of service when the minimum service required for pension entitlement was increased to 22 years, i.e. H_R=22. Since the wastage rates before H_R are lower than those after H_R, the wastage rates at 20 and 21 years total length of service in each grade with H_R=22 are replaced by the average wastage rates immediately before H_R=20. However, a sensitivity analysis on the impact of wastage rates should be performed. The results for case 2 are summarised in figures 7.8 to 7.14 and tables 7.21 to 7.28.

By comparing results in table 7.13 and table 7.21, it can be seen that when the minimum total length of service required for pension entitlement is increased from 20 years to 22 years, the total discounted cost over the 10 year planning horizon is reduced by 9%. The structure of the cost has also changed; the annual pension cost is reduced from

36.93% to 28.6% of total cost, while the stock cost is increased from 44.43% to 49.38% of total cost. The probability and expected waiting time before eventual promotion to a specified grade are affected by changes in pension entitlement policy, as can be seen from examination of tables 7.22 to 7.28. For example, from tables 7.22 and 7.23 it can be seen that 6.54% of recruits will be promoted to grade 6 in 17.88 years on average, as compared to 7.61% and 17.37 years in case 1. Other manpower policies could also be investigated in a similar way. For example, costs could be reduced by decreasing the number of staff entitled to a pension, or by further increasing the total length of service required for pension entitlement.

7.5 SUMMARY

A case study in which the decision support system developed in Chapter 6 has been used to conduct manpower planning in a military system has been described in this chapter. In the military system, individuals can only be considered for promotion after completing a specified minimum total length of service, and similarly annual pension entitlement requires completion of a specified minimum total length of service. It is also desirable that promotion rates are stable over time. The management of the military system is concerned with determining minimum cost recruitment, promotion and pension policies, which satisfy manpower requirements while ensuring that promotion rates remain stable over time and that the career prospects of individuals, measured in terms of promotion probabilities and the expected waiting time before eventual promotion, are acceptable. Since pension payments are made annually and adjusted in accordance with inflation until death, there is a substantial financial burden associated with pension policy. The demonstration decision support system has been used to evaluate the effects on both costs and career prospects of changes in policy regarding the total length of service for pension entitlement. The test results show that significant savings can be made by changing the minimum service length required for pension entitlement. This test application of the decision support system suggests that the system would be a useful aid to management in evaluating manpower policies.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 MANPOWER PLANNING

In this research, manpower planning is defined as the process of determining manpower policies which ensure that suitable numbers of qualified people are in appropriate positions at the right times in order to meet the organisational goals, while taking account of the career development opportunities of the individuals within the organisation. This definition emphasises the importance of achieving both organisational goals and the goals of individuals since the organisational goals cannot be realised without the complete support of the individuals in the organisation. The organisational goals are concerned with matching manpower requirements and supply in general, and minimising manpower costs in particular. The costs in the manpower system may comprise recruitment costs, stock costs and pension costs. Individuals are concerned with career development opportunities in terms of the likelihood of promotion and expected waiting time before promotion. In particular, stable promotion rates, defined in terms of the proportion of staff promoted, help demonstrate to individuals that their career prospects do not depend on factors, e.g. the time at which they entered the organisation, which are outside their control. Information on the probability of eventual promotion to a specified grade, and the expected waiting time to reach that grade will help management to review the manpower policies and help individuals to make decisions on whether to remain in the organisation. In cases where the probability and expected waiting time are unacceptable to the decision makers or individuals, alternative manpower policies, such as reducing the required minimum total length of service for promotion, should be developed and evaluated.

Manpower planning is considered as a process consisting of a series of stages involving the management of human resources. The process of manpower planning should not be regarded as a series of sequential stages which are carried out once and then left; these stages should be considered as interacting stages and the process should be continuous and never ending. Since the environment is dynamic and organisations are influenced by their environment, changes in manpower policies are always needed in order to adapt to the changes in the environment. Because the problems confronting manpower planners in a specific organisation at a specific time are unique, it is unlikely that a universal model which can solve all kinds of problems in manpower planning can be constructed. The type of model which is adopted must depend on the particular situation. It is only necessary to ensure that the model represents the real manpower system and can be used to evaluate appropriate manpower policies.

8.2 THE MIP MODEL DEVELOPED IN THIS RESEARCH

There has been a considerable body of work on the development and use of manpower planning models over the last three decades. Since changes on manpower policies can have significant effects on cost, cost is a useful criterion for evaluating manpower policies. This research has been concerned with developing cost minimisation models for manpower planning. Optimisation models have been developed in the past for many aspects of manpower planning, but in these models promotion rates have not been considered as decision variables, either because the importance of equalising promotion opportunities has been neglected or because of the limitations of the modelling techniques used. This research has been concerned with developing models for manpower planning which recognise the importance of promotion rate stability in hierarchical organisations. Since staff morale is likely to be affected if promotion rates vary significantly from one year to another, it is desirable that the promotion rates are as stable as possible over time. In this research a new modelling framework has been developed to determine the minimum cost recruitment, retirement and promotion policies that will satisfy the manpower needs while ensuring that promotion rates remain stable over time. The costs considered in this model consist of manpower stock costs, recruitment costs and pension costs. In the model, promotion rates are considered as decision variables, and it is required that the promotion rates are as stable as possible over time. This modelling approach results in a non-linear model because the number promoted is the product of the promotion rate and number in a grade, both of which are variables.

This non-linear manpower planning model can be modelled by using a mixed integer programming (MIP) model in which binary variables are used to represent promotion rate decisions. The requirement for stable promotion rates over time can then be modelled by imposing constraints for each grade to ensure that the promotion rate from that grade is in the same range in each year. An iterative solution procedure, in which the promotion rate range width is reduced at successive iterations, is used until either an infeasible solution is found or the range of promotion rate variation is acceptable to the decision makers. By using this approach the promotion opportunities in each year will remain as stable as required, and the minimum cost recruitment and promotion strategies can be determined.

Since the process of manpower planning is never ending and changes in manpower policies may be required in order to adapt to the changes in the environment, a tool is required to be able to evaluate policies in a changing environment. A personal computer based decision support system incorporating the MIP model has been developed. Using this system manpower policies can be generated and evaluated, and both the probability of eventual promotion to a specified grade and the expected waiting time to reach that grade can be estimated. The amount of information produced in numerical form by the decision support system may make it difficult to identify the important points in the output. In order to ease the interpretation of the output, graphical presentation is used in the decision support system. Use of this approach with representative data for a military system suggests that a decision support system of this type could be a useful tool to assist management decision making.

8.3 AREAS FOR FURTHER RESEARCH

A personal computer based decision support system has been developed in this research to help management to evaluate alternative manpower policies so that organisational goals and the goals of individuals can be satisfied. This system is based on an MIP model which is solved using an iterative procedure. However, this system is only suitable for use by those who are familiar with the mathematical models and who understand what parameters of the models should be changed when alternative policies are required. It would therefore be desirable to develop this demonstration system into a fully operational decision support system which is suitable for use by personal management.

It has been noted that effective manpower planning requires a sound information base which can reflect changes in the environment, the organisation, and individuals as soon and as accurately as possible. It has been assumed that this information base was available and that it can be used in the manpower planning decision support system developed in this research. However, in practice it would be necessary to develop a system for collecting information in a form appropriate to manpower planning so that manpower supply and requirements can be forecast, and criteria for evaluating manpower policies can be established.

The development of appropriate manpower policies involves a combination of the knowledge and practical experience of experts in manpower planning. The next stage of work in this area could be concerned with developing a knowledge-based system which will provide an efficient way to generate appropriate manpower policies and then evaluate these policies in terms of organisational goals and the goals of individuals.

TABLES

Table 3.1

Wastage Rates

	Year									
Grade	1	2	3	4	5	6	7	8	9	10
1	0.01	0.01	0.02	0.07	0.06	0.02	0.01	0.03	0.02	0.02
2	0.02	0.02	0.03	0.04	0.06	0.05	0.03	0.04	0.05	0.03
3	0.10	0.15	0.09	0.08	0.07	0.06	0.10	0.09	0.10	0.12
4	0.15	0.12	0.10	0.09	0.10	0.11	0.13	0.10	0.10	0.13
5	0.10	0.15	0.20	0.20	0.10	0.12	0.10	0.20	0.15	0.17
6	0.20	0.25	0.15	0.10	0.15	0.12	0.13	0.15	0.14	0.17

Table 3.2

Average Salary Per Person (£K/year)

	Year									
Grade	1	2	3	4	5	6	7	8	9	10
1	10	11	12	15	17	20	23	25	30	31
2	12	14	16	19	22	25	28	30	35	36
3	14	16	18	21	24	27	31	34	38	39
4	17	20	22	25	27	30	35	38	40	43
5	20	23	26	27	30	33	36	39	41	44
6	24	27	31	32	33	35	38	41	42	45

Table 3.3

Average Recruitment Cost Per Person (£K/year)

	Year									
Grade	1	2	3	4	5	6	7	8	9	10
1	20	22	24	28	32	36	42	48	52	58

Table 3.4

Parameters of the MIP Model

Grade	Initial number of staff	Target proportion of staff	Average service termination cost per person
1	1500	0.2	$2.4C_{S1t}$
2	2000	0.2	$2.7C_{S2t}$
3	2800	0.25	$3.6C_{S3t}$
4	2100	0.2	$6.0C_{S4t}$
5	1100	0.1	$12.0C_{S5t}$
6	500	0.05	$18.0C_{S6t}$

Note: 1. C_{Sit} is the average annual salary per person in grade i in year t, i=1,2,...,6, t= 1,2,...,10.
2. The target proportion of staff is assumed to be the same in each year.

Table 3.5a

The Total Cost and Range Width at Each Iteration

Iteration	Range width	Total cost (£K)
1	0.5	2387606
2	0.25	2390199
3	0.125	2392838
4	0.0625	2399094
5	0.0313	NFS

Note: NFS - no feasible solution

Table 3.5b

The Optimal Ranges at Each Iteration

	Optimal promotion rate range for grade				
Run	1	2	3	4	5
2	.25-.5	.25-.5	0-.25	0-.25	0-.25
3	.25-.375	.25-.375	.125-.25	0-.125	0-.125
4	.3125-.375	.25-.3125	.125-.1875	.0625-.125	.0625-.125

Table 3.6

Ten Possible Ranges of Equal Width for Promotion Rates

	Promotion rate range from grade				
Range	1	2	3	4	5
1	0.0000 0.0625	0.0000 0.0625	0.0000 0.0625	0.0000 0.0625	0.0000 0.0625
2	0.0625 0.1250	0.0625 0.1250	0.0625 0.1250	**0.0625** **0.1250**	**0.0625** **0.1250**
3	0.1250 0.1875	0.1250 0.1875	**0.1250** **0.1875**	0.1250 0.1875	0.1250 0.1875
4	0.1875 0.2500	0.1875 0.2500	0.1875 0.2500	0.1875 0.2500	0.1875 0.2500
5	0.2500 0.3125	**0.2500** **0.3125**	0.2500 0.3125	0.2500 0.3125	0.2500 0.3125
6	**0.3125** **0.3750**	0.3125 0.3750	0.3125 0.3750	0.3125 0.3750	0.3125 0.3750
7	0.3750 0.4375	0.3750 0.4375	0.3750 0.4375	0.3750 0.4375	0.3750 0.4375
8	0.4375 0.5000	0.4375 0.5000	0.4375 0.5000	0.4375 0.5000	0.4375 0.5000
9	0.5000 0.5625	0.5000 0.5625	0.5000 0.5625	0.5000 0.5625	0.5000 0.5625
10	0.5625 0.6250	0.5625 0.6250	0.5625 0.6250	0.5625 0.6250	0.5625 0.6250

Note: Underlined numbers are the optimal ranges at iteration 4.

Table 3.7

Manpower Cost (£K)

Year	Stock cost	Recruitment cost	Termination cost	Total cost
1	133226	14545	106816	254587
2	135823	14545	127972	278340
3	135775	14425	105146	255346
4	141089	15299	91454	247842
5	143726	15896	80669	240291
6	148588	16257	76989	241833
7	155089	17242	84947	257278
8	153237	17914	101538	272689
9	155030	17642	88012	260685
10	146531	17889	99825	264246

Table 4.1a

Results for Range Width Reduction Factor of 40%

Run	Range width	Least cost solution (£K) 3 ranges (10% overlap)	4 ranges (30% overlap)	5 ranges (50% overlap)
2	0.1	2410667	2410688	2410667
3	0.04	2441395	2424229	2441395
4	0.06	NFS	NFS	NFS

Notes: 1. At the first iteration, 4 ranges for promotion rates are used, i.e. 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, 0.75 to 1, with least cost solution £K2390199.

2. NFS - no feasible solution

Table 4.1b

Optimal Ranges for Range Width Reduction Factor of 40%

Run	Grade	Optimal promotion rate range 3 ranges	4 ranges	5 ranges
2	1	.325-.425	.275-.375	.325-.425
	2	.225-.325	.275-.375	.225-.325
	3	.1-.2	.1-.2	.1-.2
	4	.0-.1	.0-.1	.0-.1
	5	.0-.1	.0-.1	.0-.1
3	1	.355-.395	.325-.365	.355-.395
	2	.295-.335	.285-.325	.295-.335
	3	.17-.21	.15-.19	.17-.21
	4	.08-.12	.08-.12	.08-.12
	5	.08-.12	.08-.12	.08-.12

Table 4.2a

Results for 50% Reduction in Range Width Between Iterations

	Least cost solution (£K) for				
Run	Range width	2 ranges 0% overlap	3 ranges 25% overlap	4 ranges 50% overlap	5 ranges 75% overlap
2	0.125	2392838	2394035	2392838	2394035
3	0.0625	2399094	2412981	2399094	2412981
4	0.0313	NFS	2434893	NFS	2434893
5	0.0157		NFS		NFS

Notes: 1. At the first iteration, 4 ranges for promotion rates are used, i.e. 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, 0.75 to 1, with least cost solution £K2390199.
2. NFS - no feasible solution

Table 4.2b

The Optimal Ranges for 50% Reduction in Range Width Between Iterations

		Optimal Promotion Rate Range for			
Run	Grade	2 ranges	3 ranges	4 ranges	5 ranges
2	1	.25-.375	.3125-.4375	.3125-.375	.3438-.4063
	2	.25-.375	.1875-.3125	.25-.375	.1875-.3125
	3	.125-.25	.125-.25	.125-.25	.125-.25
	4	0-.125	0-.125	0-.125	0-.125
	5	0-.125	0-.125	0-.125	0-.125
3	1	.3125-.375	.3438-.4063	.3125-.375	.3438-.4063
	2	.25-.3125	.2813-.3438	.25-.3125	.2813-.3438
	3	.125-.1875	.1563-.2188	.125-.1875	.1563-.2188
	4	.0625-.125	.0625-.125	.0625-.125	.0625-.125
	5	.0625-.125	.0625-.125	.0625-.125	.0625-.125
4	1	NFS	.3282-.3594	NFS	.3282-.3594
	2		.2969-.3282		.2969-.3282
	3		.1719-.2032		.1719-.2032
	4		.0781-.1094		.0781-.1094
	5		.0781-.1094		.0781-.1094

Note: NFS - no feasible solution

Table 4.3a

Results for 60% Reduction in Range Width Between Iterations

Run	Range width	Least Cost Solution (£K) for 2 ranges 10% overlap	3 ranges 40% overlap	4 ranges 70% overlap	5 ranges 100% overlap
2	0.15	2393841	2401851	2393841	2401851
3	0.09	NFS	NFS	NFS	NFS

Notes: 1. At the first iteration, 4 ranges for promotion rates are used, i.e. 0 to 0.25, 0.25 to 0.5, 0.5 to 0.75, 0.75 to 1, with least cost solution £K2390199.
2. NFS - no feasible solution

Table 4.3b

The Optimal Ranges for 60% Reduction in Range Width Between Iterations

Run	Grade	Optimal promotion rate range for 2 ranges	3 ranges	4 ranges	5 ranges
	1	.225-.375	.3-.45	.225-.375	.3-.45
	2	.225-.375	.15-.3	.225-.375	.15-.3
2	3	0-.15	0-.15	0-.15	0-.15
	4	0-.15	0-.15	0-.15	0-.15
	5	0-.15	0-.15	0-.15	0-.15

Table 4.4a

Costs and Computational Times for Different Numbers of Overlapping Ranges - Case 1

Number of overlapping ranges	CPU time (sec.) LP	CPU time (sec.) BB	Cost (£K)
3	122	62	2429269
4	119	127	2417287
5	154	339	2416373
6	137	605	2415017
7	193	822	2414039
8	275	1907	2414201
9	227	2639	2413536
10	249	4305	2413545

Note: The XPRESS-MP software solves a mixed integer programming problem in two stages. First, the LP relaxation is solved and then Branch-and-Bound (BB) is used to solve the MIP problem.

Table 4.4b

Optimal Ranges Using Overlapping Ranges - Case 1

	Promotion from grade				
J^*	1	2	3	4	5
3	.3281-.3594	.2813-.3125	.1406-.1719	.0781-.1094	.0781-.1094
4	.3333-.3646	.2812-.3125	.1458-.1771	.0729-.1042	.0625-.0938
5	.3281-.3594	.2813-.3125	.1328-.1641	.0781-.1094	.0703-.1016
6	.3312-.3625	.2812-.3125	.1437-.1750	.0750-.1063	.0625-.0938
7	.3281-.3594	.2813-.3125	.1354-.1667	.0781-.1094	.0677-.0990
8	.3304-.3616	.2813-.3125	.1429-.1741	.0759-.1071	.0625-.0938
9	.3281-.3594	.2813-.3125	.1328-.1641	.0781-.1094	.0664-.0977
10	.3264-.3576	.2812-.3125	.1424-.1736	.0764-.1076	.0625-.0938

Note: J^* is the number of the overlapping ranges.

Table 4.5a

Costs and Computational Time for Different Numbers of Overlapping Ranges - Case 2

Number of overlapping ranges	CPU Time (sec.) LP	CPU Time (sec.) BB	Cost (£K)
4	69	44	2426256
5	138	338	2429545
6	146	290	2418175
7	109	499	2413685
8	134	1267	2415250
9	140	1686	2415836
10	150	2362	2414849

Note: The XPRESS-MP software solves a mixed integer programming problem in two stages. First, the LP relaxation is solved and then Branch-and-Bound (BB) is used to solve the MIP problem.

Table 4.5b

Optimal Ranges Using Overlapping Ranges - Case 2

	Promotion from grade				
J^*	1	2	3	4	5
4	.3386-.3698	.2865-.3177	.1615-.1927	.0677-.0989	.0677-.0989
5	.3282-.3594	.2813-.3125	.1407-.1719	.0782-.1094	.0782-.1094
6	.3344-.3656	.2782-.3094	.1532-.1844	.0719-.1031	.0594-.0906
7	.3282-.3594	.2657-.2969	.1407-.1719	.0782-.1094	.0677-.0989
8	.3327-.3639	.2836-.3148	.1496-.1808	.0737-.1049	.0558-.0870
9	.3282-.3594	.2891-.3203	.1407-.1719	.0782-.1094	.0703-.1015
10	.3317-.3629	.2865-.3177	.1476-.1788	.0747-.1059	.0608-.0920

Notes: J^* is defined as the number of the overlapping ranges.

Table 4.6

Range Bounds for Promotion Rates in Computational Time Experiments

	Promotion from grade				
Range	1	2	3	4	5
1	.2969-.3282	.2657-.2970	.1407-.1720	.0468-.0781	.0468-.0781
2	.3126-.3439	.2813-.3126	.1563-.1876	.0625-.0938	.0625-.0938
3	.3282-.3595	.2970-.3283	.1720-.2033	.0781-.1094	.0781-.1094
4	.3438-.3751	.3126-.3439	.1876-.2189	.0938-.1251	.0938-.1251
5	.3595-.3908	.3283-.3596	.2033-.2346	.1094-.1407	.1094-.1407

Table 4.7

CPU Times for Different Values of M

	CPU time (sec.)	
M value	Linear programming phase	Branch and bound phase
175	259	390
200	173	397
300	192	491
400	191	219
500	205	261
1000	229	221
1500	176	122
2000	188	186
2500	148	173
3000	102	152
3500	187	165
4000	170	147
5000	163	197
6000	104	192
7000	72	185
8000	139	231
9000	163	253
10000	171	189
20000	101	185
30000	101	184
40000	101	184
50000	101	185
60000	101	185
100000	101	185

Table 4.8

Ranges Bound for Experiments with Narrower Overall Range

	Promotion from grade				
Range	1	2	3	4	5
1	.3204-.3517	.2813-.3126	.1329-.1642	.0703-.1016	.0625-.0938
2	.3243-.3556	.2852-.3165	.1368-.1681	.0724-.1055	.0664-.0977
3	.3282-.3595	.2892-.3205	.1407-.1720	.0782-.1095	.0703-.1016
4	.3321-.3634	.2931-.3244	.1446-.1759	.0821-.1134	.0743-.1056
5	.3360-.3673	.2970-.3283	.1485-.1798	.0860-.1173	.0782-.1095

Table 4.9

CPU Times for Experiments with Narrower Overall Range

	CPU time (second)	
M value	Linear programming phase	Branch and bound phase
44	286	2358
1500	199	688
4000	194	703
100000	102	635

Table 4.10

Computational Times for Priorities Based on Grade

Experiment	Priorities in promotion from grade 1	2	3	4	5	CPU time (sec.) for BB phase
1	5	4	3	2	1	166
2	1	2	3	4	5	641
3	5	4	1	2	3	331
4	5	4	3	1	2	163
5	5	3	4	1	2	152
6	4	3	5	1	2	130
7	4	5	3	1	2	145
8	3	4	5	1	2	115
9	3	5	4	1	2	120
10	3	4	5	2	1	117
11	3	5	4	2	1	156

Note: BB - Branch and bound phase

Table 4.11

Computational Times for Priorities Based on Range

Experiment	Priorities in range 1	2	3	4	5	CPU time (sec.) for BB phase
12	1	2	3	4	5	1273
13	5	4	3	2	1	593
14	5	4	1	2	3	636
15	5	3	1	2	4	1676
16	5	4	3	1	2	592
17	5	3	4	1	2	373
18	4	5	3	1	2	295
19	4	3	5	1	2	335
20	3	5	4	1	2	472
21	3	4	5	1	2	377
22	4	5	3	2	1	295

Note: BB - Branch and bound phase

Table 4.12

Computational Times for Priorities Based on Grade and Range

Experiment	Range	Priorities in promotion from grade					CPU time (sec.) for BB phase
		1	2	3	4	5	
23	1	14	19	24	4	9	120
	2	15	20	25	5	10	
	3	13	18	23	3	8	
	4	11	16	21	1	6	
	5	12	17	22	2	7	
24	1	14	19	24	4	9	120
	2	15	20	25	5	10	
	3	13	18	23	3	8	
	4	12	17	22	2	7	
	5	11	16	21	1	6	
25	1	14	19	24	4	9	120
	2	15	20	25	5	10	
	3	13	18	23	3	8	
	4	11	16	21	1	6	
	5	11	16	21	1	6	
26	1	11	16	21	1	6	117
	2	15	20	25	5	10	
	3	11	16	21	1	6	
	4	11	16	21	1	6	
	5	11	16	21	1	6	
27	1	6	8	10	2	4	104
	2	6	8	10	2	4	
	3	6	8	10	2	4	
	4	5	7	9	1	3	
	5	5	7	9	1	3	

Note: BB - Branch and bound phase

Table 4.13

Computational Times Using Special Ordered Sets of Type 1

Weights in range					CPU time (second)	
1	2	3	4	5	Linear programming phase	Branch and bound phase
1	2	3	4	5	166	421
0	1	2	3	4	142	582
1	10	20	30	31	215	417
10	11	100	101	200	137	429
0	1	1000	2000	3000	146	513
5	4	3	2	1	150	416
3000	2000	1000	1	0	166	529

Table 7.1

Number of Staff Initially

Total length of service	Grade 1	2	3	4	5	6	Total	Percentage (%)
0	2507						2507	8.37
1	154	2457					2611	8.72
2		2659					2659	8.88
3		2587	731				3318	11.08
4		327	1957				2284	7.62
5		74	1789				1863	6.22
6		45	1329				1374	4.59
7			874	541			1415	4.72
8			492	1027			1519	5.07
9			214	1032			1246	4.16
10			107	872			979	3.27
11			57	692	358		1107	3.70
12			40	422	482		944	3.15
13				408	631		1039	3.47
14				242	760	56	1058	3.53
15				170	520	113	803	2.68
16				144	414	189	747	2.49
17				68	294	229	591	1.97
18				37	174	223	434	1.45
19				25	143	233	401	1.34
20				22	89	199	310	1.03
21					74	158	232	0.77
22					31	108	139	0.46
23					29	93	122	0.41
24					23	73	96	0.32
25						58	58	0.19
26						45	45	0.15
27						58	58	0.19
Total	2661	8149	7590	5702	4022	1835	29959	
Percentage	8.88	27.20	25.33	19.03	13.43	6.13		100

Table 7.2

Parameters of the Model

Grade	Minimum total length of service for promotion (H_{Pi})	Minimum total length of service (H_{Li})	Retirement age (A_{Ri})	Maximum total length of service at retirement (H_{Ui})
1	1	0	25	2
2	3	1	30	7
3	7	3	36	13
4	11	7	44	21
5	14	11	48	25
6	-	14	51	28

Table 7.3

Wastage Rates

Total length of service	Grade					
	1	2	3	4	5	6
1	.0031					
2	.0766	.1029				
3		.1157				
4		.1365	.0781			
5		.1012	.0624			
6		.1263	.0936			
7		.1503	.0742			
8			.0824	.0265		
9			.0504	.0377		
10			.3014	.1820		
11			.0899	.0840		
12			.1223	.1847	.1341	
13			.1650	.1192	.1004	
14				.0741	.0866	
15				.1502	.1625	.0612
16				.0659	.1021	.0240
17				.0778	.0823	.0470
18				.0791	.0816	.0320
19				.0309	.0400	.0295
20				.5137	.3676	.1741
21				.5439	.2261	.1631
22					.2932	.1537
23					.2500	.2393
24					.2768	.2667
25					.5065	.2010
26						.2961
27						.2812
28						.5983

Table 7.4

The Probabilities of Survival

Age	Probability	Age	Probability	Age	Probability
23	.99837	44	.99544	65	.97699
24	.99833	45	.99508	66	.97472
25	.99828	46	.99468	67	.97219
26	.99823	47	.99425	68	.96936
27	.99819	48	.99375	69	.96619
28	.99817	49	.99320	70	.96265
29	.99816	50	.99262	71	.95870
30	.99814	51	.99206	72	.95429
31	.99810	52	.99153	73	.94936
32	.99803	53	.99100	74	.94386
33	.99792	54	.99043	75	.93773
34	.99778	55	.98979	76	.93091
35	.99761	56	.98906	77	.92332
36	.99742	57	.98824	78	.91487
37	.99723	58	.98733	79	.90549
38	.99703	59	.98630	80	.89508
39	.99628	60	.98516	81	.88355
40	.99660	61	.98387	82	.87080
41	.99635	62	.98243	83	.85673
42	.99607	63	.98082	84	.84119
43	.99577	64	.97901	85	.0

Table 7.5

The Optimal Ranges at Each Iteration

Run	Range width	Optimal range for grade				
		1	2	3	4	5
1	.5	.5-1	0-.5	0-.5	0-.5	0-.5
2	.25	.75-1	.25-.5	.25-.5	0-.25	0-.25
3	.125	.875-1	.25-.375	.375-.5	.125-.25	0-.125
4	.0625	.875-.9375	.3125-.375	.375-.4375	.1875-.25	.0625-.0983
5	.0313	.9063-.9375	.3483-.375	.375-.4063	.2188-.25	.0625-.0938
6	.0156	.9219-.9375	.3594-.375	.375-.3906	.2188-.2344	.2781-.0937

Table 7.6

Number of Staff at End of Year 9 - Case 1

Total length of service	Grade 1	2	3	4	5	6	Total	Percentage (%)
0	2567						2567	9.81
1	152	2396					2548	9.74
2		2287					2287	8.74
3		1187	874				2061	7.88
4		554	1195				1749	6.69
5		309	1414				1723	6.59
6		152	1382				1534	5.87
7			797	540			1409	5.39
8			338	674			1012	3.87
9			230	913			1143	4.37
10			76	868			944	3.61
11			50	714	213		977	3.74
12			33	600	459		1092	4.18
13				345	476		833	3.18
14				212	418	40	670	2.56
15				113	286	61	460	1.76
16				92	297	88	477	1.82
17				74	308	124	506	1.93
18				44	241	119	404	1.54
19				28	201	155	384	1.47
20				7	142	187	336	1.28
21					100	173	275	1.05
22					82	189	271	1.04
23					63	164	227	0.87
24					32	117	149	0.57
25						100	112	0.43
26						65	65	0.25
27						38	38	0.15
Total	2719	6885	6389	5224	3318	1620	26155	
Percentage	10.40	26.32	24.43	19.97	12.69	6.19		100

Table 7.7

Number of Staff at End of Year 10 - Case 1

Total length of service	Grade 1	2	3	4	5	6	Total	Percentage (%)
0	2143						2143	8.40
1	152	2407					2559	10.03
2		2289					2289	8.98
3		1165	858				2023	7.93
4		580	1251				1831	7.18
5		290	1329				1619	6.35
6		154	1398				1552	6.09
7			797	540			1409	5.53
8			420	837			1257	4.93
9			189	781			970	3.80
10			71	836			907	3.56
11			39	635	190		864	3.39
12			24	445	341		810	3.18
13				410	544		969	3.80
14				244	473	37	754	2.96
15				134	364	71	569	2.23
16				81	259	82	422	1.65
17				64	269	107	440	1.73
18				52	275	144	471	1.85
19				33	222	135	390	1.53
20				7	117	144	268	1.05
21					101	168	271	1.06
22					63	154	217	0.85
23					55	150	205	0.80
24					41	126	167	0.65
25						97	110	0.43
26						71	71	0.28
27						46	46	0.18
Total	2295	6885	6376	5099	3314	1532	25501	
Percentage	9.00	27.00	25.00	20.00	13.00	6.01		100

Table 7.8

Number of Promotions at End of Year 10 - Case 1

Total length of service	Grade 1	2	3	4	5
1	2407				
2	140				
3		858			
4		445			
5		208			
6		116			
7		57	540		
8			311		
9			132		
10			90		
11			30	190	
12			19	156	
13			13	131	
14				75	37
15				46	33
16				25	22
17				20	23
18				16	24
19				10	19
20				6	16
21				2	11
22					8
23					6
24					5
25					3
Total	2547	1684	1135	677	207

Table 7.9

Promotion Rates at End of Year 10 - Case 1

Total length of service	Grade 1	2	3	4	5
1	.9377				
2	.9211				
3		.3752			
4		.3749			
5		.3755			
6		.3754			
7		.3750	.3907		
8			.3902		
9			.3905		
10			.3913		
11			.3947	.2189	
12			.3800	.2185	
13			.3939	.2183	
14				.2174	.0777
15				.2170	.0789
16				.2212	.0769
17				.2174	.0774
18				.2162	.0779
19				.2273	.0788
20				.2143	.0796
21				.2857	.0775
22					.0800
23					.0732
24					.0794
25					.0937
Average	.9367	.3751	.3906	.2186	.0782

Table 7.10

Average Promotion Rates over 10 Years - Case 1

Total length of service	Grade 1	2	3	4	5
1	.9323				
2	.9224				
3		.3701			
4		.3676			
5		.3665			
6		.3667			
7		.3678	.3790		
8			.3792		
9			.3789		
10			.3775		
11			.3755	.2218	
12			.3777	.2232	
13			.3801	.2228	
14				.2252	.0820
15				.2263	.0851
16				.2261	.0861
17				.2272	.0859
18				.2278	.0899
19				.2247	.0898
20				.2273	.0912
21				.2264	.0875
22					.0845
23					.0815
24					.0845
25					.0791
Average	.9317	.3686	.3788	.2236	.0864

Table 7.11

Number of Losses by Wastage at End of Year 10 - Case 1

Total length of service	Grade 1	2	3	4	5	6	Total
1	8						8
2	12	247					259
3		265					265
4		162	68				230
5		56	75				131
6		39	132				171
7		23	103				126
8			66	14			80
9			17	25			42
10			69	166			235
11			7	73			80
12			6	132	29		167
13			5	72	46		123
14				26	41		67
15				32	68	2	102
16				7	29	1	37
17				7	24	4	35
18				6	25	4	35
19				1	10	4	15
20				14	74	27	115
21				4	32	30	66
22					29	27	56
23					21	45	66
24					17	44	61
25					16	24	40
26						30	30
27						18	18
28						23	23
Total	20	792	548	579	461	283	2683

Table 7.12

Number of Retirements - Case 1

	Grade						
Year	1	2	3	4	5	6	Total
1	0	22	18	5	10	23	78
2	0	19	13	2	8	13	55
3	0	44	13	2	6	12	77
4	0	177	9	2	8	12	208
5	0	96	11	3	8	12	130
6	0	82	11	3	7	11	114
7	0	76	9	2	8	14	109
8	0	58	12	2	10	15	97
9	0	72	12	2	12	15	113
10	0	72	15	2	13	15	117
Total	0	718	123	25	90	142	1098

Table 7.13

Manpower Cost (£K) - Case 1

Year	Stock	Recruitment	Lump sum	Annual pension	Total
1	509148	42541	167999	328007	1047695
2	508767	53379	168335	311633	1042114
3	509255	53830	162683	349697	1075465
4	507344	54949	158573	404450	1125316
5	504635	52138	154065	432256	1143094
6	502041	55068	151611	470761	1179481
7	497325	54545	156931	488750	1197551
8	494185	55136	152856	472014	1174191
9	491360	55916	152826	467444	1167546
10	485207	47111	151737	438131	1122186
Total	5009267	524613	1577616	4163143	11274639
%	44.43	4.65	13.99	36.93	100

Table 7.14

The Probability of Eventual Promotion from Grade 1 - Case 1

Total length of service	Eventual promotion to grade 2	3	4	5	6
0	.9919	.6519	.4357	.2286	.0761
1	.9224	.6712	.4485	.2354	.0784

Table 7.15

The Expected Waiting Time (years) for Promotion from Grade 1 - Case 1

Total length of service	Eventual promotion to grade 2	3	4	5	6
0	1.06	3.86	8.02	12.92	17.37
1	1.00	2.86	7.02	11.92	16.37

Table 7.16

The Probability of Eventual Promotion from Grade 2 - Case 1

Total length of service	Eventual promotion to grade 3	4	5	6
1	.6528	.4362	.2289	.0762
2	.7277	.4863	.2552	.0849
3	.6954	.4957	.2605	.0867
4	.6610	.4969	.2621	.0873
5	.5532	.4355	.2315	.0771
6	.3678	.2818	.1544	.0515

Table 7.17

The Expected Waiting Time (years) for promotion from Grade 2 - Case 1

Total length of service	Eventual promotion to grade			
	3	4	5	6
1	2.86	7.02	11.92	16.37
2	1.86	6.02	10.92	15.37
3	1.75	5.06	9.92	14.37
4	1.60	4.13	8.92	13.37
5	1.34	3.28	7.94	12.38
6	1.00	2.88	6.98	11.39

Table 7.18

The Probability and Expected Waiting Time of Eventual Promotion from Grade 3 - Case 1

Total length of service	Probability Promotion to grade			Expected Waiting Time (years) promotion to grade		
	4	5	6	4	5	6
3	.6252	.3274	.1090	4.99	9.91	14.37
4	.6781	.3552	.1182	3.99	8.91	13.37
5	.7233	.3788	.1261	2.99	7.91	12.37
6	.7980	.4179	.1391	1.99	6.91	11.37
7	.7662	.4198	.1400	1.88	5.98	10.39
8	.7188	.4200	.1407	1.75	5.12	9.44
9	.5955	.3837	.1296	1.58	4.39	8.54
10	.6790	.4323	.1504	1.60	4.39	7.87
11	.5677	.3767	.1304	1.33	4.09	7.21
12	.3801	.2535	.0851	1.00	3.69	6.67

Table 7.19

The Probability and Expected Waiting Time of Eventual Promotion from Grade 4 - Case 1

Total length of service	Probability Promotion to grade 5	Probability Promotion to grade 6	Expected Waiting Time (years) Promotion to grade 5	Expected Waiting Time (years) Promotion to grade 6
7	.4971	.1650	5.83	10.34
8	.5106	.1695	4.83	9.34
9	.5307	.1762	3.83	8.34
10	.6487	.2154	2.83	7.34
11	.6150	.2148	2.77	6.57
12	.6617	.2327	2.78	5.99
13	.6670	.2238	2.69	5.67
14	.6305	.2029	2.55	5.30
15	.6483	.1916	2.42	4.94
16	.5963	.1593	2.18	4.53
17	.5311	.1261	1.91	4.14
18	.4376	.0925	1.59	3.77
19	.2859	.0610	1.21	3.32
20	.2264	.0433	1.00	2.96

Table 7.20

The Probability and Expected Waiting Time of Eventual Promotion from Grade 5 to Grade 6 - Case 1

Total length of service	Probability	Expected waiting time (years)
11	.2987	5.82
12	.3450	4.82
13	.3835	3.82
14	.3626	3.58
15	.3688	3.37
16	.3483	3.10
17	.3154	2.78
18	.2722	2.49
19	.2097	2.23
20	.2189	2.18
21	.1914	1.96
22	.1718	1.72
23	.1350	1.37
24	.0791	1.00

Table 7.21

Manpower Cost (£K) - Case 2

Year	Stock	Recruitment	Lump sum	Annual pension	Total
1	511279	42541	180546	191647	926013
2	511263	50293	182712	181674	925942
3	513856	54141	180069	203132	951198
4	514820	55882	180886	229600	981188
5	512903	50476	179945	268140	1011464
6	510707	53839	178383	313818	1056747
7	506742	54777	183513	350216	1095248
8	501243	53022	176627	393811	1124703
9	496593	55523	172120	410372	1134608
10	488620	47134	169344	392906	1055004
Total	5068026	474628	1784145	2935316	10262115
%	49.38	4.63	17.39	28.60	100

Table 7.22

The Probability of Eventual Promotion from Grade 1 - Case 2

Total length of service	Eventual promotion to grade				
	2	3	4	5	6
0	.9917	.6412	.4179	.2051	.0654
1	.9224	.6601	.4303	.2112	.0673

Table 7.23

The Expected Waiting Time (years) for Promotion from Grade 1 - Case 2

Total length of service	Eventual promotion to grade				
	2	3	4	5	6
0	1.06	3.90	8.13	13.23	17.88
1	1.00	2.90	7.13	12.23	16.88

Table 7.24

The Probability of Eventual Promotion from Grade 2 - Case 2

Total length of service	Eventual promotion to grade			
	3	4	5	6
1	.6420	.4185	.2054	.0654
2	.7156	.4665	.2289	.0729
3	.6816	.4734	.2327	.0742
4	.6445	.4714	.2326	.0741
5	.5349	.4090	.2034	.0649
6	.3516	.2609	.1337	.0427

Table 7.25

The Expected Waiting Time (years) for promotion from Grade 2 - Case 2

Total length of service	Eventual promotion to grade			
	3	4	5	6
1	2.90	7.13	12.23	16.88
2	1.90	6.13	11.23	15.88
3	1.78	5.17	10.24	14.88
4	1.61	4.23	9.24	13.88
5	1.34	3.39	8.26	12.89
6	1.00	2.98	7.31	11.91

Table 7.26

The Probability and Expected Waiting Time of Eventual Promotion from Grade 3 - Case 2

Total length of service	Probability Promotion to grade			Expected Waiting Time (years) Promotion to grade		
	4	5	6	4	5	6
3	.6083	.2980	.0949	5.09	10.23	14.88
4	.6599	.3232	.1030	4.09	9.23	13.88
5	.7038	.3447	.1098	3.09	8.23	12.88
6	.7765	.3803	.1212	2.09	7.23	11.88
7	.7422	.3803	.1214	1.98	6.31	10.91
8	.6943	.3785	.1213	1.82	5.46	9.96
9	.5753	.3439	.1111	1.64	4.74	9.07
10	.6493	.3833	.1270	1.63	4.69	8.41
11	.5370	.3286	.1084	1.34	4.37	7.76
12	.3501	.2143	.0688	1.00	3.97	7.22

Table 7.27

The Probability and Expected Waiting Time of Eventual Promotion from Grade 4 - Case 2

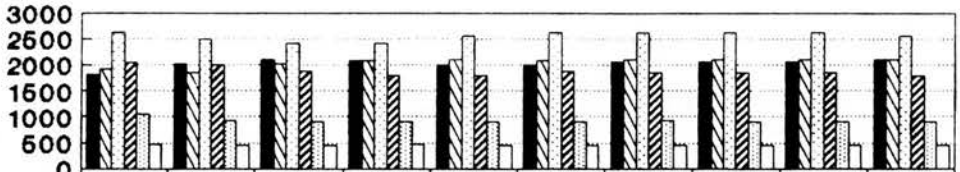
Total length of service	Probability Promotion to grade		Expected Waiting Time (years) Promotion to grade	
	5	6	5	6
7	.4618	.1467	6.12	10.83
8	.4744	.1507	5.12	9.83
9	.4930	.1566	4.12	8.83
10	.6026	.1914	3.12	7.83
11	.5716	.1900	3.06	7.09
12	.6118	.2047	3.06	6.53
13	.6121	.1966	2.97	6.22
14	.5761	.1787	2.82	5.83
15	.5887	.1697	2.68	5.44
16	.5407	.1428	2.43	4.98
17	.4864	.1152	2.15	4.50
18	.4139	.0856	1.82	4.01
19	.2998	.0543	1.43	3.48
20	.1734	.0266	1.00	2.94

Table 7.28

The Probability and Expected Waiting Time of Eventual Promotion from Grade 5 to Grade 6 - Case 2

Total length of service	Probability	Expected waiting time (years)
11	.2841	6.16
12	.3281	5.16
13	.3648	4.16
14	.3463	3.97
15	.3568	3.77
16	.3424	3.51
17	.3186	3.20
18	.2884	2.88
19	.2415	2.53
20	.2014	2.22
21	.1535	1.94
22	.1488	1.72
23	.1165	1.36
24	.0645	1.00

FIGURES



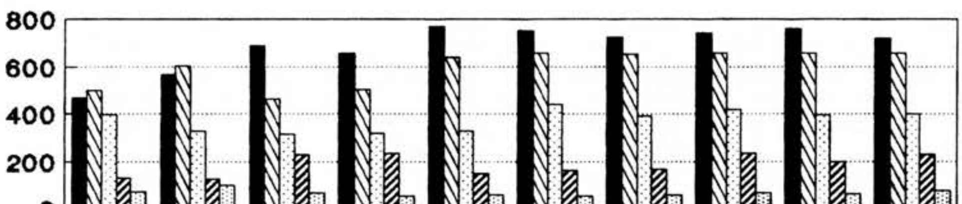
	1	2	3	4	5	6	7	8	9	10
I	1816	2031	2100	2097	2002	2011	2065	2063	2060	2100
II	1929	1856	2026	2094	2100	2090	2100	2100	2100	2100
III	2625	2506	2431	2424	2564	2625	2625	2625	2625	2565
IV	2049	2003	1887	1800	1800	1879	1860	1862	1871	1800
V	1050	924	900	900	900	900	919	900	900	900
VI	471	450	450	461	450	452	450	450	450	450

Year

Grade

I II III IV V VI

Figure 3.1 Number of staff



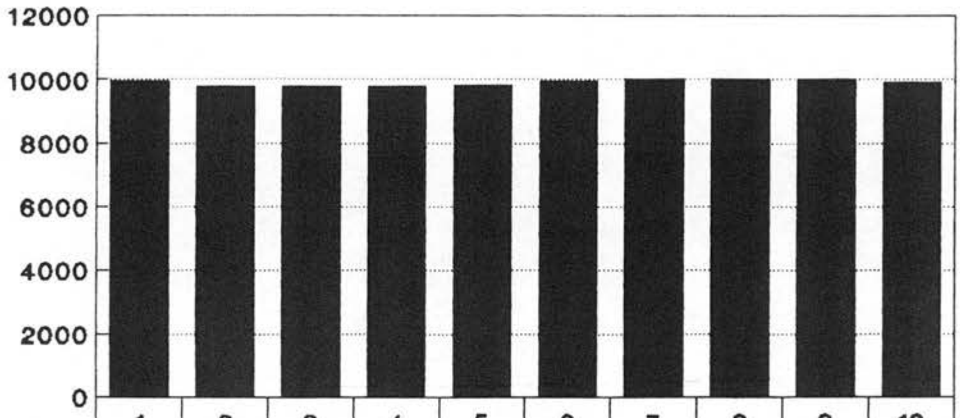
	1	2	3	4	5	6	7	8	9	10
I	469	568	690	656	769	751	726	740	761	719
II	500	603	464	506	637	656	653	656	656	656
III	395	328	313	319	328	441	391	420	394	402
IV	131	128	229	236	148	164	165	233	198	230
V	71	97	68	56	58	56	57	68	63	77

Year

Grade

I II III IV V

Figure 3.2 Number of Promotions

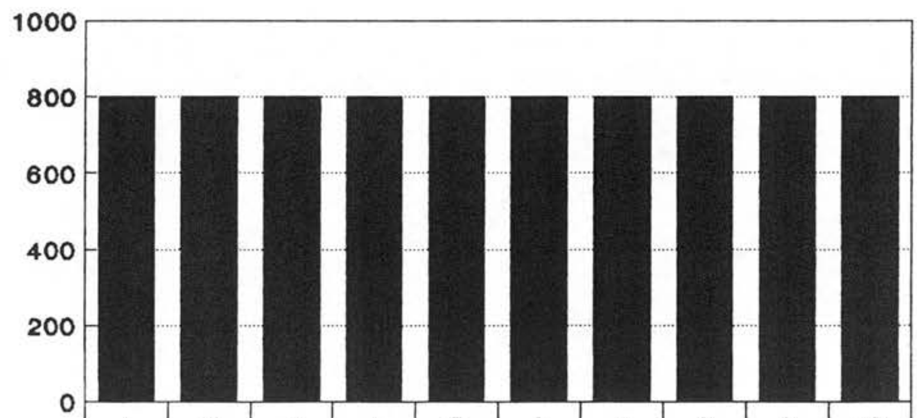


Year	1	2	3	4	5	6	7	8	9	10
Staff	9940	9768	9794	9777	9816	9957	10019	9999	10007	9916

Year

Staff

Figure 3.3 Total Staff



Year	1	2	3	4	5	6	7	8	9	10
Recruits	800	800	800	800	800	800	800	800	800	800

Year

Recruits

Figure 3.4 Number of Recruits

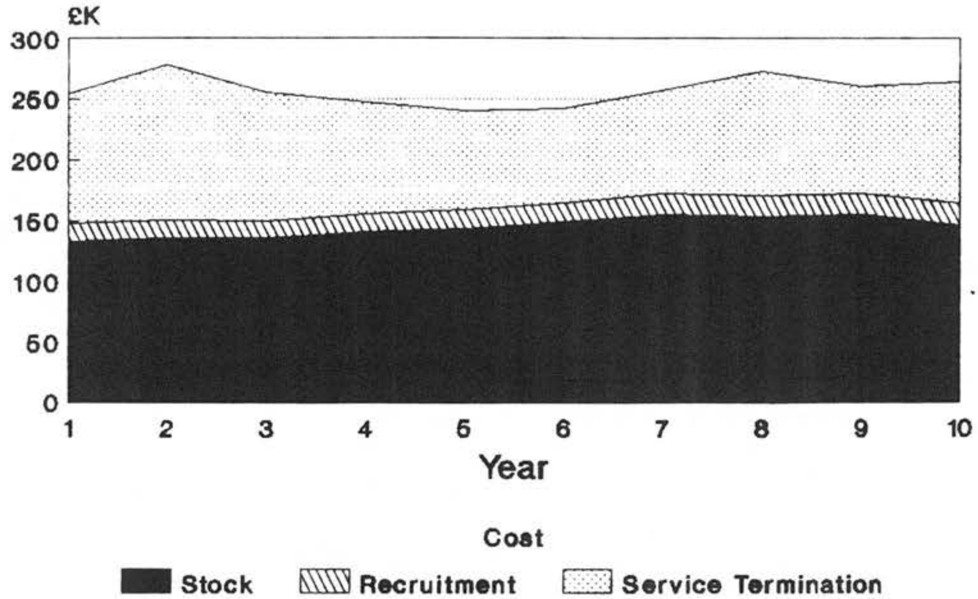


Figure 3.5 Manpower Costs

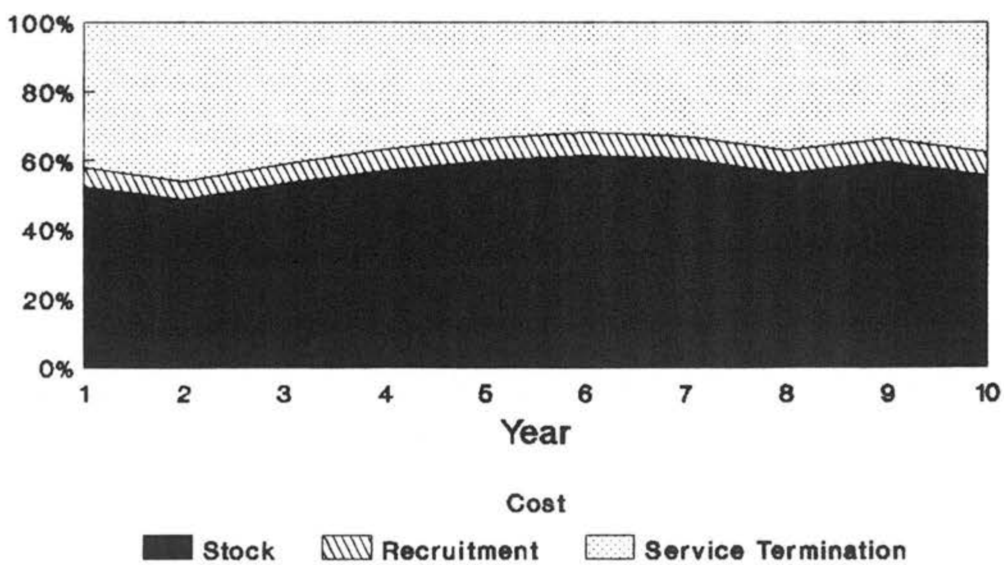
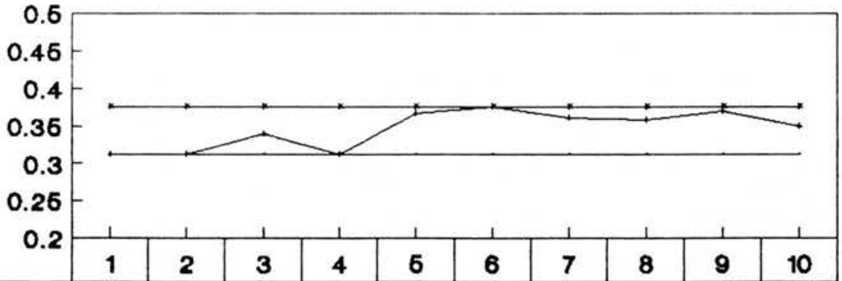


Figure 3.6 Distribution of Manpower Costs



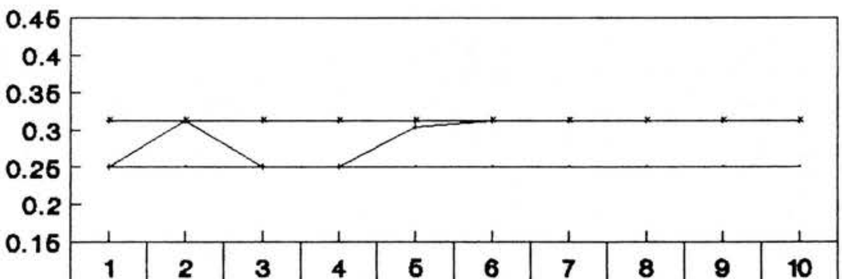
	1	2	3	4	5	6	7	8	9	10
Lower Bound	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312
Projection	0.312	0.312	0.34	0.312	0.367	0.375	0.361	0.358	0.369	0.349
Upper Bound	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375	0.375

Year

Rate

Lower Bound — Projection — Upper Bound

Figure 3.7 Promotion Rates
grade 1 to grade 2



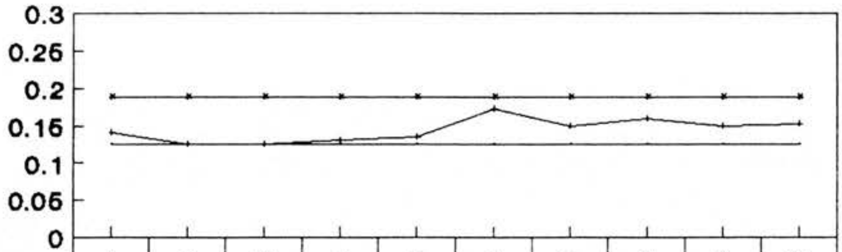
	1	2	3	4	5	6	7	8	9	10
Lower Bound	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
Projection	0.25	0.312	0.25	0.25	0.304	0.312	0.312	0.312	0.312	0.312
Upper Bound	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312	0.312

Year

Rate

Lower Bound — Projection — Upper Bound

Figure 3.8 Promotion Rates
grade 2 to grade 3



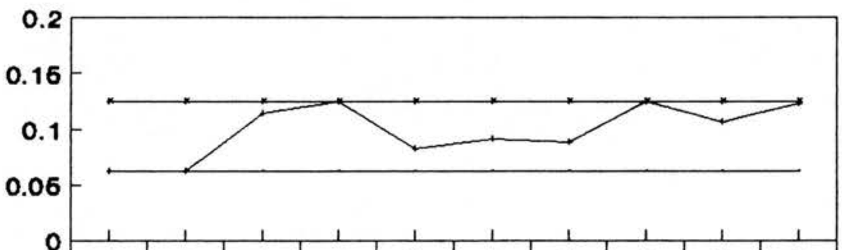
	1	2	3	4	5	6	7	8	9	10
Lower Bound	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125
Projection	0.141	0.125	0.125	0.131	0.135	0.172	0.149	0.16	0.15	0.153
Upper Bound	0.188	0.188	0.188	0.188	0.188	0.188	0.188	0.188	0.188	0.188

Year

Rate

Lower Bound — Projection — Upper Bound

Figure 3.9 Promotion Rates grade 3 to grade 4



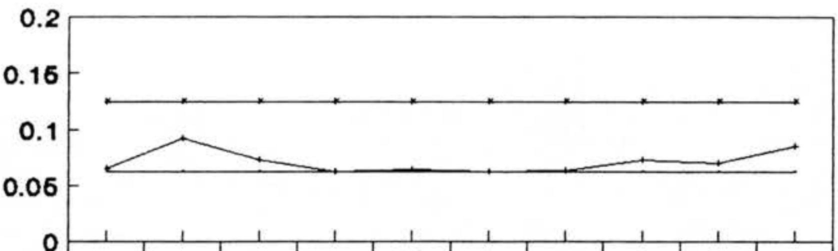
	1	2	3	4	5	6	7	8	9	10
Lower Bound	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
Projection	0.062	0.062	0.114	0.125	0.082	0.091	0.088	0.125	0.106	0.123
Upper Bound	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

Year

Rate

Lower Bound — Projection — Upper Bound

Figure 3.10 Promotion Rates grade 4 to grade 5



	1	2	3	4	5	6	7	8	9	10
Lower Bound	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062	0.062
Projection	0.065	0.092	0.073	0.062	0.064	0.062	0.063	0.073	0.07	0.085
Upper Bound	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

Year

Rate

Lower Bound — Projection — Upper Bound

Figure 3.11 Promotion Rates
grade 5 to grade 6

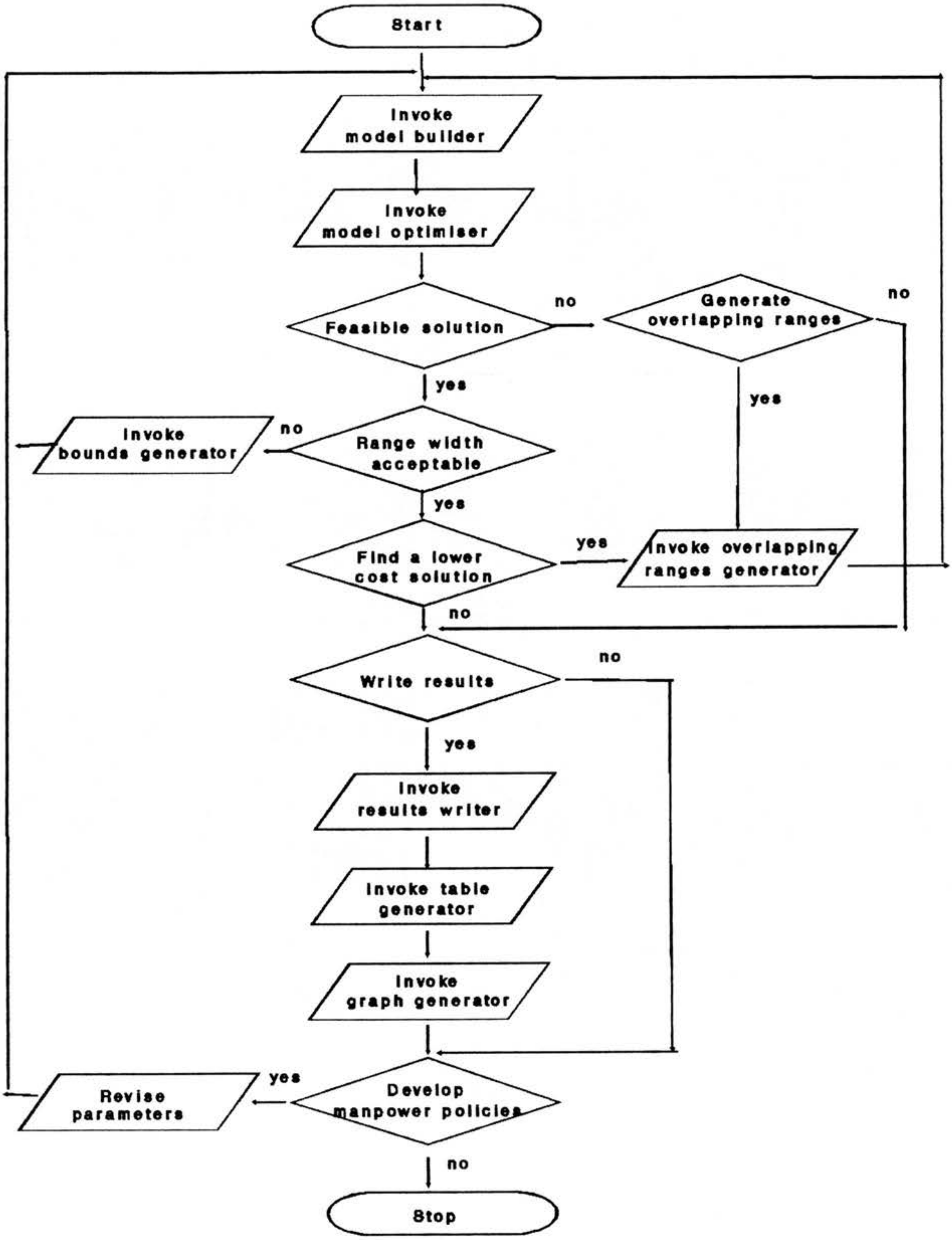
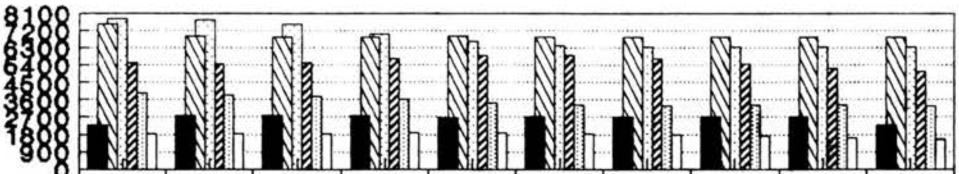


Figure 6.1 Procedure of the Decision Support System



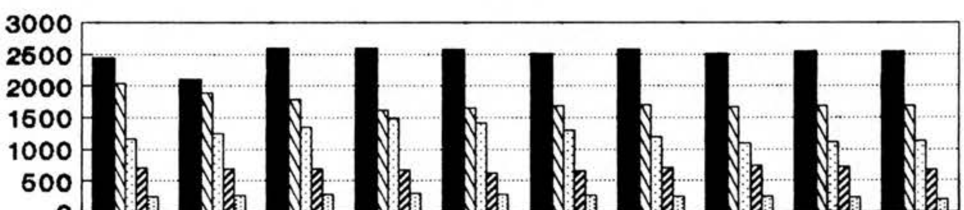
	1	2	3	4	5	6	7	8	9	10
I	2295	2777	2772	2800	2684	2749	2707	2727	2719	2295
II	7571	6918	6885	6885	6941	6886	6885	6884	6885	6885
III	7820	7787	7548	7038	6643	6448	6375	6376	6389	6376
IV	5545	5479	5537	5762	5938	5939	5739	5450	5224	5099
V	3958	3875	3784	3640	3485	3362	3315	3319	3318	3314
VI	1803	1817	1844	1864	1859	1828	1764	1699	1620	1532

Year

Grade

I II III IV V VI

Figure 7.1 Number of Staff - Case 1



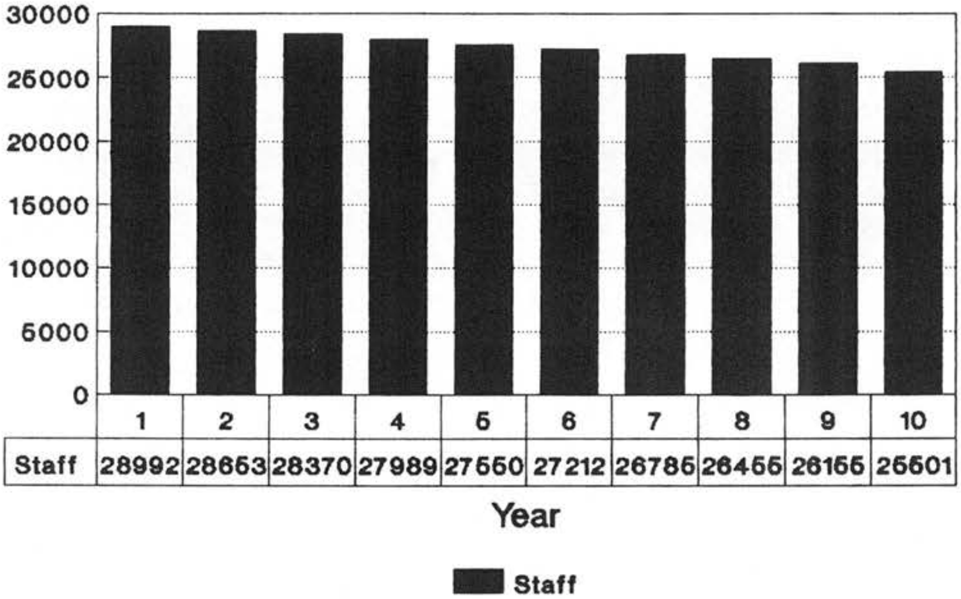
	1	2	3	4	5	6	7	8	9	10
I	2453	2115	2601	2597	2582	2514	2575	2516	2555	2547
II	2047	1891	1789	1621	1645	1688	1708	1668	1692	1684
III	1167	1241	1346	1488	1423	1291	1195	1097	1120	1135
IV	707	686	698	672	628	656	711	742	721	677
V	248	272	293	299	279	266	245	245	230	207

Year

Grade

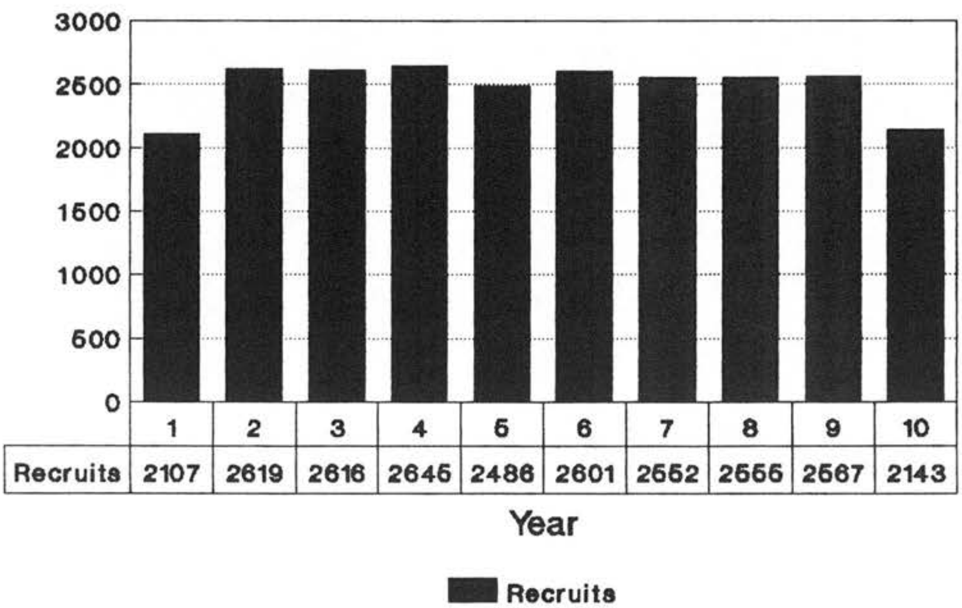
I II III IV V

Figure 7.2 Number of Promotions - Case 1



Year	1	2	3	4	5	6	7	8	9	10
Staff	28992	28653	28370	27989	27550	27212	26785	26455	26155	25501

Figure 7.3 Total Staff - Case 1



Year	1	2	3	4	5	6	7	8	9	10
Recruits	2107	2619	2616	2645	2486	2601	2552	2555	2567	2143

Figure 7.4 Number of Recruits - Case 1

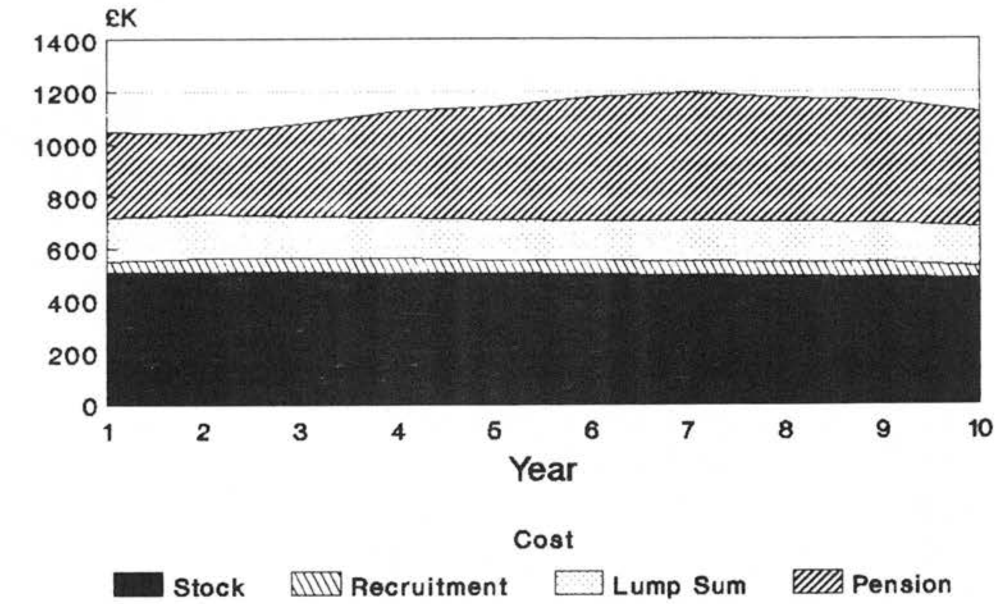


Figure 7.5 Manpower Costs - Case 1

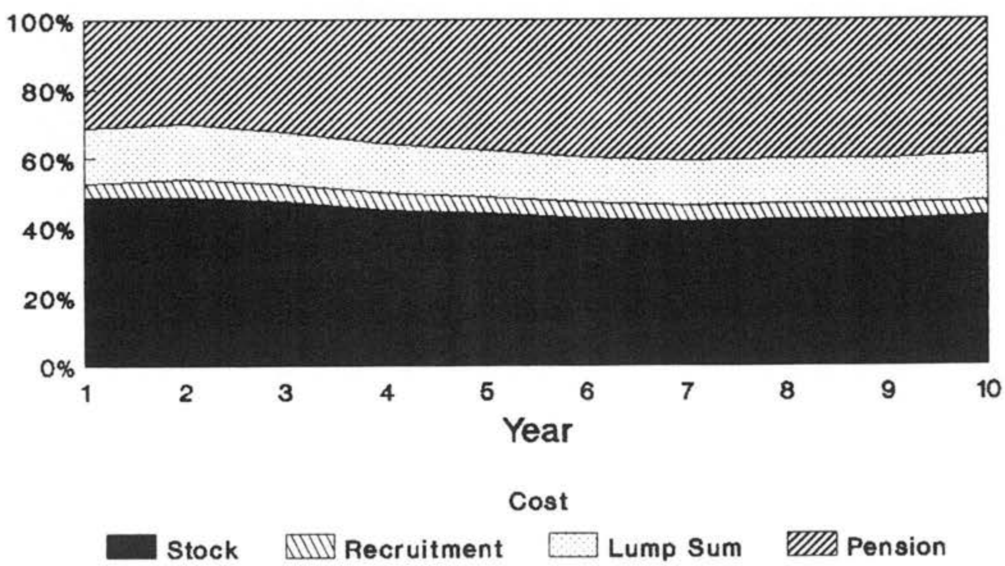
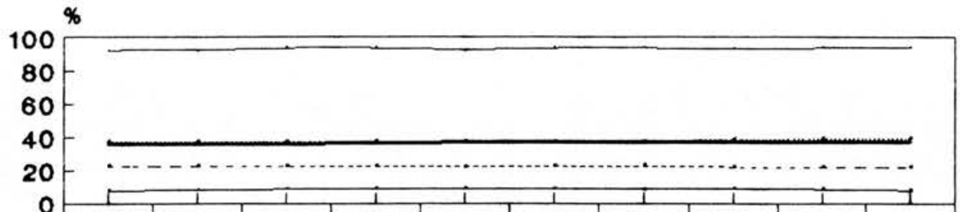


Figure 7.6 Distribution of Manpower Costs - Case 1



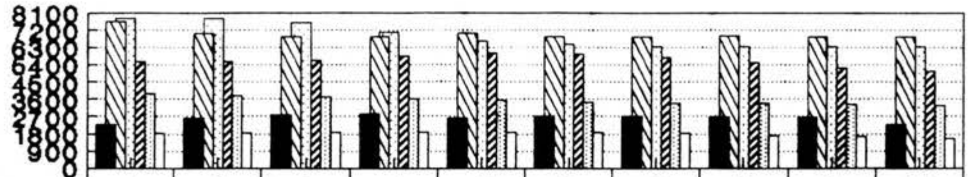
	1	2	3	4	5	6	7	8	9	10
I	92.2	92.2	93.7	93.7	92.2	93.7	93.7	92.9	93.7	93.7
II	36	36	36	36.6	37.1	37.5	37.5	37.5	37.5	37.5
III	37.5	37.5	37.5	37.5	37.5	37.5	37.5	38.8	39	39.1
IV	22.8	22.7	22.6	22.3	22.1	22.5	23	21.9	21.9	21.9
V	7.8	8.3	8.7	9.3	9.1	8.9	8.6	9.1	8.8	7.8

Year

Grade

I II III IV V

Figure 7.7 Promotion Rates - Case 1



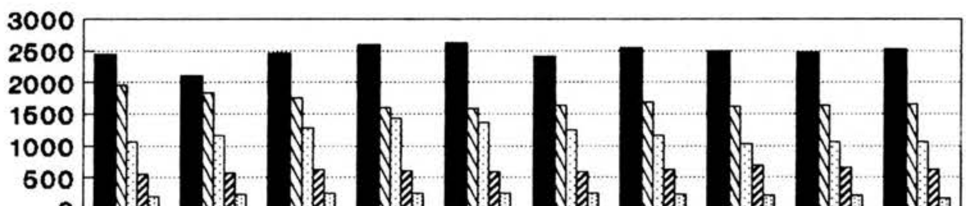
	1	2	3	4	5	6	7	8	9	10
I	2295	2625	2778	2846	2609	2724	2714	2649	2695	2295
II	7658	7046	6884	6886	7031	6897	6886	6909	6885	6886
III	7827	7809	7586	7102	6670	6457	6376	6375	6376	6376
IV	5613	5585	5651	5862	6015	6005	5831	5507	5258	5101
V	3886	3775	3721	3651	3561	3463	3394	3376	3337	3315
VI	1815	1842	1872	1884	1883	1861	1813	1740	1646	1530

Year

Grade

I II III IV V VI

Figure 7.8 Number of Staff - Case 2



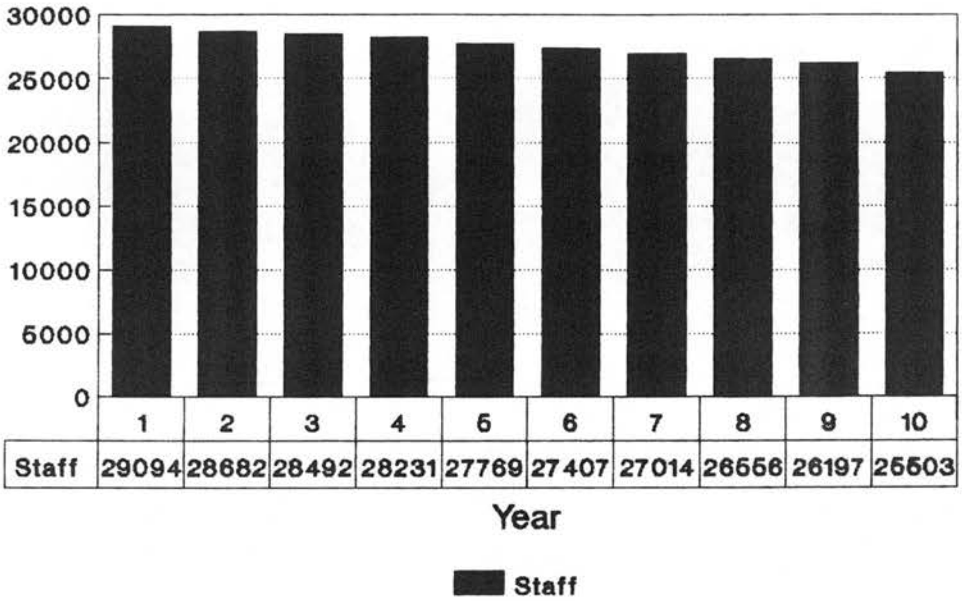
	1	2	3	4	5	6	7	8	9	10
I	2453	2115	2459	2602	2624	2405	2551	2502	2482	2525
II	1955	1838	1755	1602	1579	1636	1682	1617	1634	1647
III	1071	1168	1284	1429	1374	1240	1156	1034	1058	1066
IV	551	568	617	610	583	595	630	684	664	627
V	208	236	250	251	246	246	237	221	214	177

Year

Grade

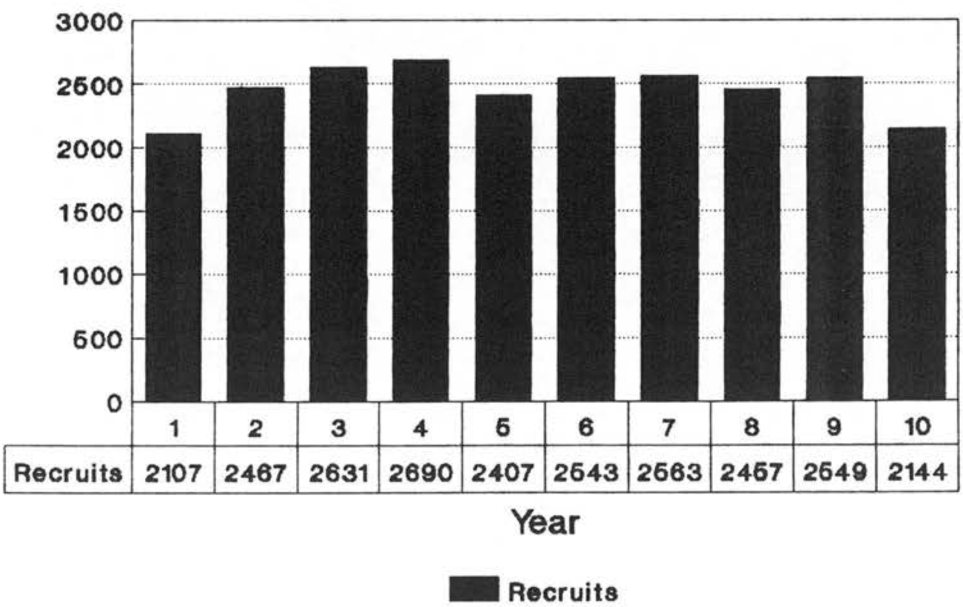
I II III IV V

Figure 7.9 Number of Promotions - Case 2



	1	2	3	4	5	6	7	8	9	10
Staff	29094	28682	28492	28231	27769	27407	27014	26556	26197	25503

Figure 7.10 Total Staff - Case 2



	1	2	3	4	5	6	7	8	9	10
Recruits	2107	2467	2631	2690	2407	2543	2563	2457	2549	2144

Figure 7.11 Number of Recruits - Case 2

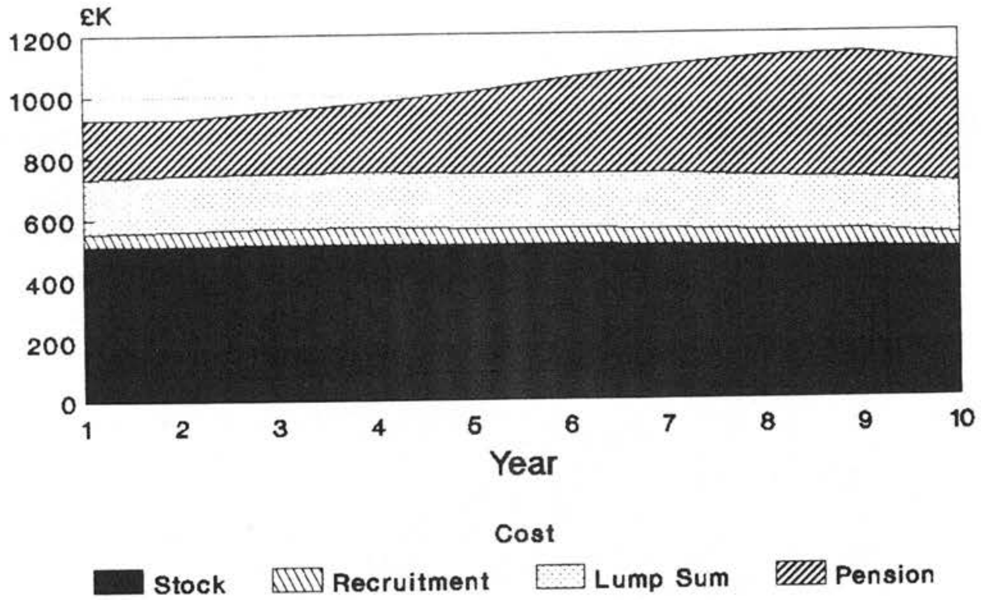


Figure 7.12 Manpower Costs - Case 2

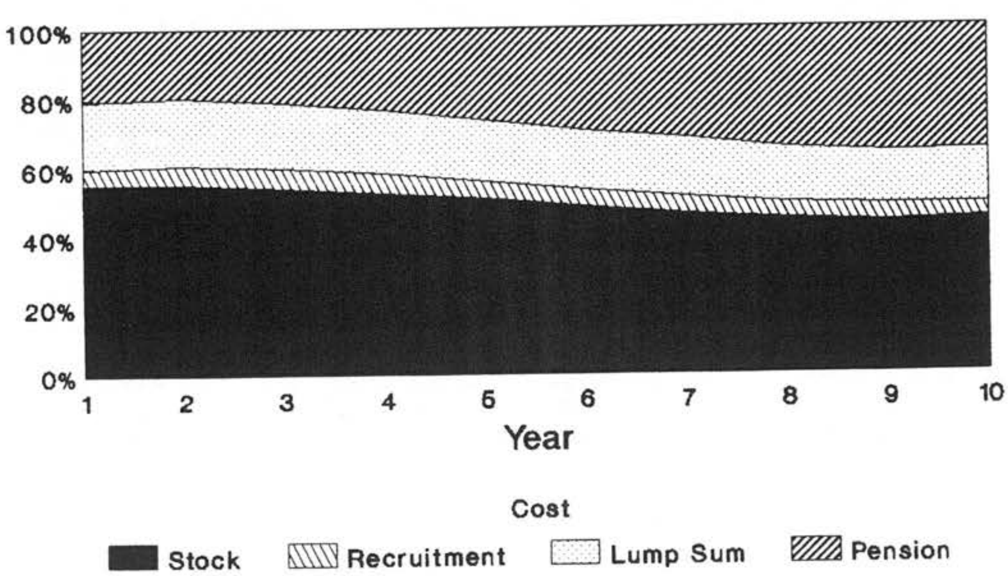
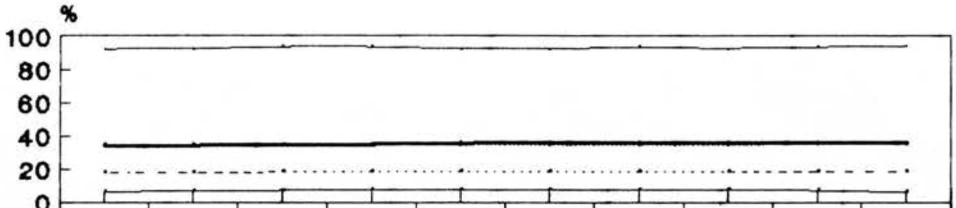


Figure 7.13 Distribution of Manpower Costs - Case 2



	1	2	3	4	5	6	7	8	9	10
I	92.2	92.2	93.7	93.7	92.2	92.2	93.6	92.2	93.7	93.7
II	34.3	34.4	34.4	35	35.7	35.9	36	35.9	35.9	36
III	34.4	34.3	34.4	34.7	34.7	34.4	34.4	34.5	35.6	35.2
IV	17.8	17.9	18.5	18.5	18.6	18.6	18.6	18.7	18.7	18.7
V	6.5	7.2	7.4	7.7	7.8	7.8	7.8	7.6	7.6	6.4

Year

Grade

I II III IV V

Figure 7.14 Promotion Rates - Case 2

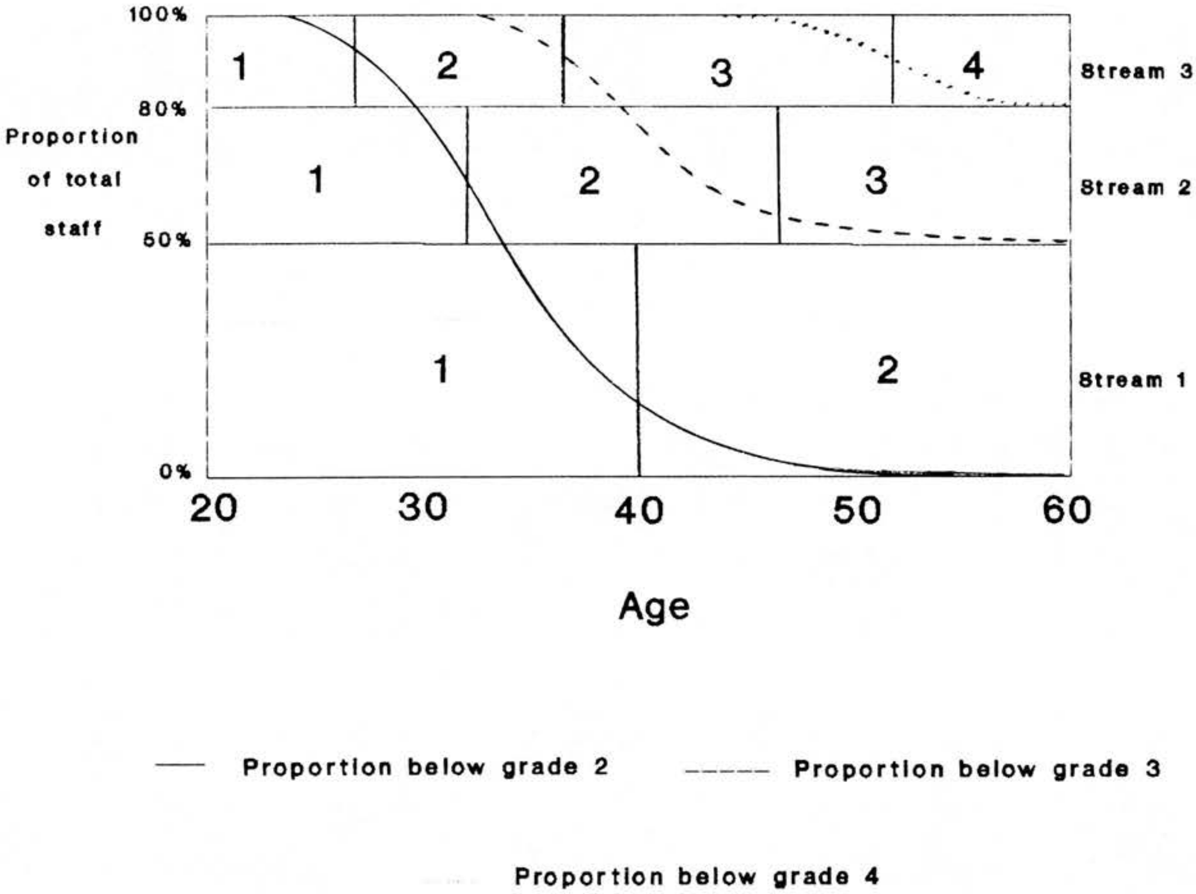


Diagram 1 Career Streams Diagram

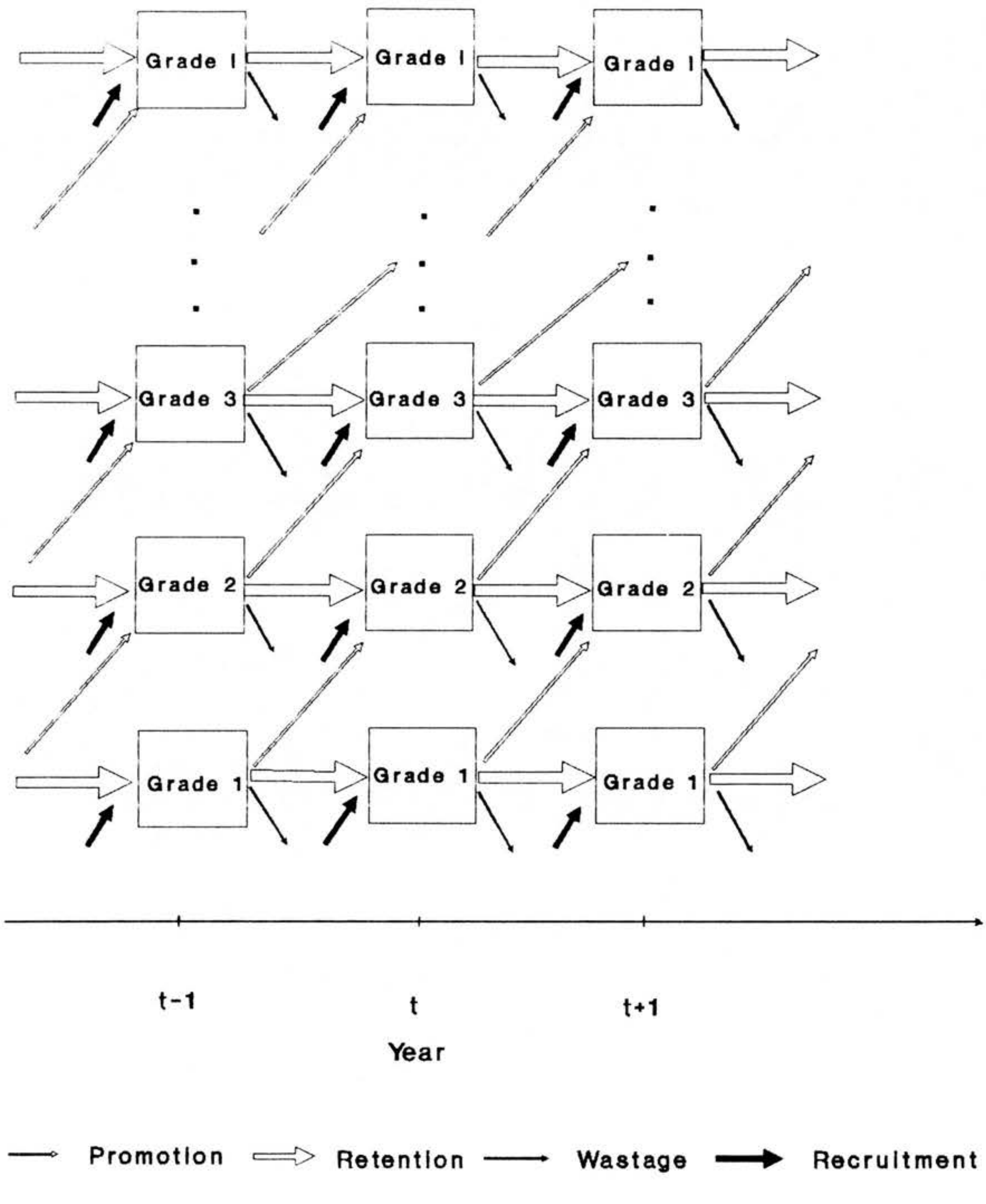


Diagram 2 Stocks and Flows for a Manpower System

BIBLIOGRAPHY

Alam, M. A. (1984), "A Career Structure Model for IAF Officers". Indian Air Force Journal 3, 6-16.

Alam, M. A. (1985a), "Steady State Career Structure Model for Indian Army Officers". Defence Management 12, 34-42.

Alam, M. A. (1985b), "Seniority Distributions in Markov Manpower Models". J. of IIM 13, 1-10.

Andersen, D. F. and Emmerichs, R. M. (1982), "Analyzing U.S. Military retirement policies". Simulation, November, 151-158.

Anthony, S. and Wilson, J. (1990), "Manpower Modelling Using a Spreadsheet". J. of Mgmt Sci. 18, 505-510.

Ashford, R. W. and Daniel, R. C. (1992), "Some Lessons in Solving Practical Integer Programs". Journal of the Operational Research Society 43, 425-433.

Bartholomew, D. J. (1963), "A Multi-Stage Renewal Process". J. of the Royal Statistical Society B25, 150-168.

Bartholomew, D. J. (1974), "Stochastic Models and their Application: a Review", in Clough, Lewis and Oliver, eds. (1974), 81-90.

Bartholomew, D. J. (1977), "Maintaining a Grade or Age Structure in a Stochastic Environment". Adv. Appl. Prob. 9, 1-17.

Bartholomew, D. J. (1982), Stochastic Models for Social Processes. 3rd ed. Chichester: John Wiley & Sons.

Bartholomew, D. J. and Forbes, A. F. (1981), Statistical Techniques for Manpower Planning. 2nd ed. Chichester: John Wiley & Sons.

Bartholomew, D. J. and Smith, A. R., eds. (1971), Manpower and Management Science. London: The English Universities Press Ltd.

Belton, V. and O'keefe, R. M., eds. (1986), Recent Developments in Operational Research. Oxford: Pergamon Press.

Billington, J. N. (1987), "Visual Interactive Modelling and Manpower Planning". European J. of Opl. Res. 30, 77-84.

Boller, R. L., Lehto, R. K., Offir, J. and Silverman, J. (1978), "Design and Use of a Force Structure Simulation Model", in Charnes, Cooper and Niehaus, eds. (1978), 173-191.

Bowey, A. M. (1974), A Guide to Manpower Planning. London: Macmillan.

Brynat, D. T. and Niehau, R. J. eds. (1978), Manpower Planning and Organisation Design. New York: Plenum.

Butler, A. D. (1974), "A System for Equalizing Promotion Rates between Grades within Similar Hierarchies", in Clough, Lewis and Oliver, eds. (1974), 205-217.

Camm, J. D., Raturi, A. S. and Tsubakitani, S. (1990), "Cutting Big M Down to Size". Interfaces 20, No. 5, 61-66.

Campbell, M. (1990), Harvard Graphics 2.3. Berkeley: McGraw-Hill.

Charness, A., and Cooper, W. W. (1961), Management Models and Industrial Applications of Linear Programming. Chichester: John Wiley & Sons.

Charness, A., Cooper, W. W. and Niehaus, R. J. (1971), "A Generalized Network Model for Training and Recruiting Decisions in Manpower Planning", in Bartholomew and Smith, eds. (1971), 115-130.

Charnes, A., Cooper, W. W. and Niehaus, R. J., eds. (1978), TIMS Studies in the Management Sciences. Vol. 8. Amsterdam: North-Holland Publishing Company.

Charnes, A., Cooper, W. W., Niehaus, R. J. and Scholtz, D. (1974), "Multi-Level Models for Career Management and Resource Planning", in Clough, Lewis and Oliver, eds. (1974), 91-112.

Charnes, A., Cooper, W. W., Lewis, K. A. and Niehaus, R. J. (1978), "A Multi-Level Coherence Model for EEO Planning", in Charnes, Cooper and Niehaus, eds. (1978), 13-29.

Clark, T. D. and Lawson, K. (1984), "The Rotation and Assignment of Imbalanced Air Force Specialties: a Policy Analysis Model". Simulation, July, 41-48.

Clough, D. J., Dudding, R. C. and Price, W. L. (1970), "Mathematical Programming Models of a Quasi-Independent Subsystem of the Canadian Forces Manpower System", in Smith, ed. (1970), 299-315.

Clough, D. J., Lewis, C. G. and Oliver, A. L., eds. (1974), Manpower Planning Models. London: The English Universities Press Ltd.

Collins, R. W., Gass, S. I. and Rosendahl, E. E. (1983), "The ASCAR Model for Evaluating Military Manpower Policy". Interfaces 13, No. 3 44-53.

Creegan, J. B. and Monforte, F. A. (1990), "Southern California Gas Company Uses Special Ordered Sets to Model Regulatory Guidelines". Interfaces 20, No. 4, 28-42.

Dash Associates, (1991), XPRESS-MP Reference Manual. Northants: Dash Associates Ltd.

Davies, G. S. (1973), "Structural Control in a Graded Manpower System". Management Science 20, 76-84.

Davies, G. S. (1975), "Maintainability of Structures in Markov Chain Models under Recruitment Control". Journal of Applied Probability 12, 376-382.

Davies, G. S. (1976), "Consistent Recruitment in a Graded Manpower System". Management Science 22, 1215-1220.

Department of Employment, (1974), Company Manpower Planning. London: HMSO.

Edwards, J., Leek, C., Loveridge, R., Lumley, R., Mangan, J. and Silver, M. eds. (1983), Manpower Planning. Chichester: John Wiley & Sons.

Edwards, J. (1983), "Theory and Practice - Where Do They Meet?", in Edwards, Leek, Loveridge, Lumley, Mangan and Silver eds. (1983), 197-203.

Edwards, J. (1986), "Manpower Planning in 1986", in Belton and O'keefe, eds. (1986), 27-48.

Eiger, A., Jacobs, J. M., Chung, D. B. and Selsor, J. L. (1988), "The US Army's Occupational Specialty Manpower Decision Support System". Interfaces 18, No. 1, 57-73.

Forbes, A. F. (1971), "Markov Chain Models for Manpower System", in Bartholomew and Smith, eds. (1971), 93-113.

Forbes, A. F. (1974), Some Statistical Models and Techniques for Manpower Planning. Doctoral Thesis, University of Kent, Canterbury.

Forrest, J. J. H., Hirst, J. P. H. and Tomlin, J. A. (1974), "Practical Solution of Large Mixed Integer Programming Problems with UMPIRE". Management Science 20, 736-773.

Gass, S. I. (1986), "A Process for Determining Priorities and Weights for Large-Scale Linear Goal Programmes". J. Opl Res. Soc. 37, 779-785.

Gass, S. I., Collins, R. W., Meinhardt, C. W., Lemon, D. M. and Gillette, M. D. (1988), "The Army Manpower Long-range Planning System". Operations Research 36, 5-17.

Glen, J. J. (1977), "Length of Service Distributions in Markov Manpower Models". Opl. Res. Q. 28, 975-982.

Glen, J. J. (1991), "A Mixed Integer Programming Model of a Multicohort Single-Species Fishery". IMA Journal of Mathematics Applied in Business & Industry 3, 95-106.

Gohl, J. and Opelland, H. J. (1978), "A Methodological Review of the Treatment of Individual Goals in Some Recent German System Approaches towards Manpower Planning", in Bryant and Niehaus (1978), 57-72.

Grinold, R. C. (1976), "Manpower Planning with Uncertain Requirements". Operations Research 24, 387-399.

Grinold, R. C. and Marshall, K. T. (1977), Manpower Planning Models. Amsterdam: North-Holland.

Grinold R. C. and Stanford, R. E. (1974), "Optimal Control of a Graded Manpower System". Management Science 20, 1201-1216.

Groover, R. O. (1970), "Persym: a Generalized Entity Simulation Model of a Military Personnel System", in Smith, ed. (1970), 153-166.

Hartley, R. V. (1976), Operations Research: a Managerial Emphasis. Englewood Cliffs: Prentice-Hall Inc.

Holz, B. W. and Wroth, J. M. (1980), "Improving Strength Forecasts: Support for Army Manpower Management". Interfaces 10, No. 6, 37-52.

Jones, D. G. (1978), "A Method of Re-interpreting Promotion Rates in Terms of Promotion Prospectuses". Appl. Statist. 27, 58-66.

Jones, E. (1964), "An Actuarial Problem Concerning the Royal Marines". Journal of the Institute of Actuaries Students Society 6, 38-42.

Keenay, G. A. (1974), Manpower Planning in large Organizations - a Statistical Approach. Doctoral Thesis, University of Cambridge, Cambridge.

Keenay, G. A. and Morgan, R. W. (1979), "Model for Recruitment Planning in a Company". Personnel Review 8, 5-9.

Keenay, G. A., Morgan, R. W. and Ray, K. H. (1974), "The Use of Steady-State Models for Career Planning in an Expanding Organization", in Clough, Lewis and Oliver, eds. (1974), 151-159.

Keenay, G. A., Morgan, R. W. and Ray, K. H. (1977a), "The Camel Model: a Model for Career Planning in a Hierarchy". Personnel Review 6, 43-50.

Keenay, G. A., Morgan, R. W. and Ray, K. H. (1977b), "An Analytical Model for Company Manpower Planning". Opl. Res. Q. 28, 983-995.

Lee, R. A. and Piper, J. (1988), "Dimensions of Promotion Culture in Midland Bank". Personnel Review 17, 15-24.

Leeson, G. W. (1979), "Wastage in a Hierarchical Manpower System". J. Opl. Res. Soc. 30, 341-348.

Malloch, H. (1986), "Manpower Modelling with Computer Spreadsheet". Personnel Management, May, 44-46.

Malloch, H. (1988), "Evaluating Strategies on a Cost-Based Manpower Planning Model". Personnel Review 17, 22-28.

McClean, S. (1986), "Extending the Entropy Stability Measure for Manpower Planning". J. Opl Res. Soc. 37, 1133-1138.

McClean, S. and Abodunde, T. (1978), "Entropy as a Measure of Stability in a Manpower System". J. Opl Res. Soc. 29, 885-889.

Mehlmann, A. (1980), "An Approach to Optimal Recruitment and Transition Strategies for Manpower System Using Dynamic Programming". J. Opl Res. Soc. 31, 1009-1015.

Morgan, R. W. (1970), "Manpower Planning in the Royal Air Force: an Exercise in Linear Programming", in Smith, ed. (1970), 317-325.

Morgan, R. W. (1971), "The Use of a Steady State Model to Obtain the Recruitment, Retirement and Promotion Policies of an Expanding Organisation", in Bartholomew and Smith, eds. (1971), 283-291.

Morgan, R. W. (1979), "Some Models for a Hierarchical Manpower System". J. Opl Res. Soc. 30, 727-736.

Niehaus, R. J., Sholtz, D. and Thompson, G. L. (1978), "Managerial Tests of Conversational Manpower Planning Models", in Charnes, Cooper and Niehaus, eds. (1978), 153-171.

Parsons, R. (1974), Statistics Analysis. New York: Harper & Row Publishers.

Perry, W. D. (1978), "Corporate and Management Policies for Recruitment, Employment, Training, and Retention of the Secondary Labour Force", in Charnes, Cooper and Niehaus, eds. (1978), 31-60.

Phillips, D. T., Ravindran, A. and Solberg, J. J. (1976), Operations Research: Principles and Practice. New York: John Wiley & Sons.

Price, W. L. and Piskor, W. G. (1972), "The Application of Goal Programming to Manpower Planning". INFOR 10, 221-231.

Pugh, A. L. (1983), DYNAMO User's Manual. Cambridge: MIT Press.

Purkiss, C. J. (1970), "Models for Examming and Optimizing Manpower Deployment", in Smith, ed. (1970), 265-284.

Rao, P. P. (1990), "A Dynamic Programming Approach to Determine Optimal Manpower Recruitment Policies". J. Opl Res. Soc. 41, 983-988.

Ray, K. (1977), "Managerial Manpower Planning - a Systematic Approach". Long Range Planning 10, 21-30.

Robinson, D. (1974), "Two-Stage Replacement Strategies and their Application to Manpower Planning". Management Science 21, 199-208.

Silverman, J., Steuer, R. E. and Whisman, A. W. (1988), "A Multi-period, Multiple Criteria Optimization System for Manpower Planning". European Journal of Operational Research 34, 160-170.

Smith, A. R. ed. (1970), Models of Manpower Systems. London: The English Universities Press Ltd.

Smith, A. R. (1974), "Somes Problems of Manpowe System Analysis in the UK Civil Service", in Clough, Lewis and Oliver, eds. (1974), 183-203.

Smith, A. R. ed. (1976), Manpower Planning in the Civil Service. London: Her Majesty's Stationery Office.

Smith, A. R. and Bartholomew, D. J. (1988), "Manpower Planning in the United Kingdom: An Historical Review". J. Opl Res. Soc. 39, 235-248.

Stanford, R. E. (1980), "The effects of Promotion by Seniority in Growth-Constrained Organizations". Management Science 26, 680-693.

Stanford, R. E. (1985), "A Stochastic Model of Personnel Mobility: Application of Reliability Theory in Manpower System Analysis". Operations Research 33, 109-125.

Steuer, R. E. and Choo, E. U. (1983), "An Interactive Weighted Tchebycheff Procedure for Multiple Objective Programming". Mathematical Programming 26, 326-344.

Steuer, R. E. (1986), Multiple Criteria Optimization: Theory, Computation, and Application. New York: John Wiley & Sons.

Tyler, G. W. (1986), "Organization Size and Staff Tenure". J. Opl Res. Soc. 37, 373-380.

Vajda, S. (1978), Mathematics of Manpower Planning. Chichester: John Wiley & Sons.

Venema, W. and Wessels, J. (1988), "Systematic Modelling and Model Handling for Manpower Planning Systems". Decision Support System 4, 503-508.

Verhoeven, C. J. (1982), Techniques in Corporate Manpower Planning. Boston: Kluwer· Nijhoff Publishing.

Walmsley, C. W. (1971), "A Simulation Model for Manpower Management", in Bartholomew and Smith, eds. (1971), 163-170.

Wijngaard, J. (1983), "Aggregation in Manpower Planning". Management Science 29, 1427-1435.

Wishart, D. (1976a), "Manpower Supply Models III: The KENT Model", in Smith, ed. (1976a), 124-139.

Wishart, D. (1976b), "Manpower Supply Models IV: The MANSIM Model", in Smith, ed. (1976b), 140-165.

Woodward, M. (1983), "On Forecasting Grade, Age and Length of Service Distributions in Manpower Systems". J. R. Statist. Soc. A 146, 74-84.

APPENDIX A - THE VARIABLES AND PARAMETERS OF THE MIP MODEL (3-18)

Variables

n_{it} number of staff in grade i, i=1,2,....,I, at end of year t, t=1,2,...,T

s_t the total number of staff in the manpower system at end of year t, t=1,2,...,T

r_{it} number of recruits to grade i, i=1,2,....,I, in year t, t=1,2,....,T

m_{it} number of staff promoted from grade i, i=1,2,...I-1, to grade i+1 in year t, t=1,2,...,T

δ_{ji} binary variable of promotion rate range j, j=1,2,...,J, from grade i, i=1,2,...,I-1, to grade i+1, where δ_{ji}=1 if the jth range is chosen; otherwise, δ_{ji}=0

Model Parameters

N_{i0} number of staff in grade i, i=1,2,....,I, initially, i.e. at start of year 1

W_{it} wastage rate in grade i, i=1,2,....,I, in year t, t=1,2,....,T, as a proportion of the number of staff in post at the start of the year; the wastage rate includes loss of individuals from system for whatever reason, e.g. retirement

S_t target total number of staff in the manpower system at end of year t, t=1,2,....,T

E_{Ut} maximum upper proportional deviation in target total number of staff at end of year t, t=1,2,....,T

E_{Lt} maximum lower proportional deviation in target total number of staff at end of year t, t=1,2,....,T

F_{Uit} maximum upper proportional deviation in target number of staff in grade i,i=1,2,...,I, at end of year t, t=1,2,...,T

F_{Lit} maximum lower proportional deviation of target number of staff in grade i, i=1,2,...,I, at end of year t, t=1,2,...,T

G_{it} target proportion of staff in grade i, i=1,2,...,I, at end of year t, t=1,2,...,T

R_{Uit} upper bound on number of recruits to grade i, i=1,2,...,I, in year t, t=1,2,...,T

R_{Lit} lower bound on number of recruits to grade i, i=1,2,...,I, in year t, t=1,2,...,T

$B_{ji}^{(k)}$ upper bound of promotion rate range j, j=1,2,...,J, from grade i to grade i+1, i=1,2,...,I-1, in any year at iteration k+1, k=1,2,...

$B_{j+1,i}^{(k)}$ lower bound of promotion rate range j, j=1,2,...,J, from grade i to grade i+1, i=1,2,...,I-1, in any year at iteration k+1, k=1,2,...

C_{Sit} average annual salary per person in grade i, i=1,2,...,I, in year t, t=1,2,...,T

C_{Rit} average recruitment cost per person recruited to grade i, i=1,2,...,I, in year t, t=1,2,...,T

C_{Git} average gratuity cost per person in grade i, i=1,2,...,I, leaving the system in year t, t=1,2,...,T

α annual discount rate

APPENDIX B - THE MIP MODEL (5-8)

Minimise

$$\sum_{t=1}^{T} C_{Rt}(1+\alpha)^{-t} r_t + \sum_{i=1}^{I} \sum_{t=1}^{T} \{ \sum_{h=H_{Li}}^{A_{Ri}-A_U-2} \sum_{e=A_L+h}^{A_U+h} C_{Siht}(1+\alpha)^{-t} n_{ihet} +$$

$$\sum_{A_{Ri}-A_U-1}^{H_{Ui}-1} \sum_{e=A_L+h}^{A_{Ri}-1} C_{Siht}(1+\alpha)^{-t} n_{ihet} +$$

$$\sum_{h=H_{Li}+1}^{H_R-1} \sum_{e=A_L+h}^{A_U+h} C_{Fiht}(1+\alpha)^{-t}(W'_{ihet} n_{i,h-1,e-1,t-1}) +$$

$$\sum_{h=H_R}^{A_{Ri}-A_U-1} \sum_{e=A_L+h}^{A_U+h} \sum_{z=1}^{A_E-e} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} W'_{ihet} n_{i,h-1,e-1,t-1} +$$

$$\sum_{h=A_{Ri}-A_U}^{H_{Ui}} \sum_{e=A_L+h}^{A_{Ri}} \sum_{z=1}^{A_E-e} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} W'_{ihet} n_{i,h-1,e-1,t-1} +$$

$$\sum_{h=A_{Ri}-A_U}^{H_{Ui}} \sum_{e=A_{Ri}}^{A_{Ri}} \sum_{z=1}^{A_E-A_{Ri}} C_{Piht}(1+\beta)^{z}(1+\alpha)^{-(z+t)} Y_{e,e+z} n_{ihet} \} \qquad (5\text{-}8a)$$

subject to

$$n_{10et} - D_e r_t = 0 \qquad (5\text{-}8b1)$$

$e=A_L, A_L+1, \ldots, A_U, \forall t$

$$n_{1het} - (1 - W_{1het}) n_{1,h-1,e-1,t-1} = 0 \qquad (5\text{-}8b2)$$

$h=1,2,\ldots,H_{P1}-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h, \forall t$

$$n_{1het} - (1 - W_{1het}) n_{1,h-1,e-1,t-1} + m_{1het} = 0 \qquad (5\text{-}8b3)$$

$h=H_{P1}, H_{P1}+1, \ldots, A_{R1}-A_U-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h, \forall t$

$$n_{1het} - (1 - W_{1het}) n_{1,h-1,e-1,t-1} + m_{1het} = 0 \qquad (5\text{-}8b4)$$

$h=A_{R1}-A_U, A_{R1}-A_U+1, \ldots, H_{U1},\ e=A_L+h, A_L+h+1, \ldots, A_{R1}, \forall t$

$$n_{Ihet} - (1 - W_{Ihet}) n_{I,h-1,e-1,t-1} - \theta_{I-1,h} m_{I-1,het} = 0 \qquad (5\text{-}8b5)$$

$h=H_{LI}+1, H_{LI}+2, \ldots, A_{RI}-A_U-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,$

$\theta_{I-1,h}=0$ for $h>H_{UI-1}$, $\theta_{I-1,h}=1$ for $h\le H_{UI-1}$, $\forall t$

$$n_{Ihet} - (1 - W_{Ihet})n_{I,h-1,e-1,t-1} - \theta_{I-1,h}m_{I-1,het} = 0 \tag{5-8b6}$$

$h=A_{RI}-A_U, A_{RI}-A_U+1, \ldots, H_{UI},\ e=A_L+h, A_L+h+1, \ldots, A_{RI},$

$\theta_{I-1,h}=0$ for $h>H_{Ui-1}$, $\theta_{I-1,h}=1$ for $h\le H_{Ui-1}$, $\forall t$

$$n_{ihet} - \theta_{i-1,h}m_{i-1,het} = 0 \tag{5-8b7}$$

$i=2,3,\ldots,I,\ h=H_{Li},\ e=A_L+h, A_L+h+1, \ldots, A_U+h,$

$\theta_{i-1,h}=0$ for $h>H_{Ui-1}$, $\theta_{i-1,h}=1$ for $h\le H_{Ui-1}$, $\forall t$

$$n_{ihet} - (1 - W_{ihet})n_{i,h-1,e-1,t-1} - \theta_{i-1,h}m_{i-1,het} = 0 \tag{5-8b8}$$

$i=2,3,\ldots,I-1,\ h=H_{Li}+1, H_{Li}+2, \ldots, H_{Pi}-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,$

$\theta_{i-1,h}=0$ for $h>H_{Ui-1}$, $\theta_{i-1,h}=1$ for $h\le H_{Ui-1}$, $\forall t$

$$n_{ihet} - (1 - W_{ihet})n_{i,h-1,e-1,t-1} - \theta_{i-1,h}m_{i-1,het} + m_{ihet} = 0 \tag{5-8b9}$$

$i=2,3,\ldots,I-1,\ h=H_{Pi}, H_{Pi}+1, \ldots, A_{Ri}-A_U-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,$

$\theta_{i-1,h}=0$ for $h>H_{Ui-1}$, $\theta_{i-1,h}=1$ for $h\le H_{Ui-1}$, $\forall t$

$$n_{ihet} - (1 - W_{ihet})n_{i,h-1,e-1,t-1} - \theta_{i-1,h}m_{i-1,het} + m_{ihet} = 0 \tag{5-8bx}$$

$i=2,3,\ldots,I-1,\ h=A_{Ri}-A_U, A_{Ri}-A_U+1, \ldots, H_{Ui},\ e=A_L+h, A_L+h+1, \ldots, A_{Ri},$

$\theta_{i-1,h}=0$ for $h>H_{Ui-1}$, $\theta_{i-1,h}=1$ for $h\le H_{Ui-1}$, $\forall t$

$$s_t - \sum_{i=1}^{I} \left[\sum_{h=H_{Li}}^{A_{Ri}-A_U-2} \sum_{e=A_L+h}^{A_U+h} n_{ihet} + \sum_{h=A_{Ri}-A_U-1}^{H_{Ui}-1} \sum_{e=A_L+h}^{A_{Ri}-1} n_{ihet} \right] = 0 \tag{5-8c1}$$

$$s_t \ge S_t(1-E_{Lt}) \qquad \forall t \tag{5-8c2}$$

$$s_t \le S_t(1+E_{Ut}) \qquad \forall t \tag{5-8c3}$$

$$\sum_{h=H_{Li}}^{A_{Ri}-A_U-2} \sum_{e=A_L+h}^{A_U+h} n_{ihet} + \sum_{h=A_{Ri}-A_U-1}^{H_{Ui}-1} \sum_{e=A_L+h}^{A_{Ri}-1} n_{ihet} \ge G_{it}S_t(1 - F_{Lit}) \quad \forall i, t \tag{5-8d1}$$

$$\sum_{h=H_{Li}}^{A_{Ri}-A_U-2} \sum_{e=A_L+h}^{A_U+h} n_{ihet} + \sum_{h=A_{Ri}-A_U-1}^{H_{Ui}-1} \sum_{e=A_L+h}^{A_{Ri}-1} n_{ihet} \le G_{it}S_t(1 + F_{Uit}) \quad \forall i, t \tag{5-8d2}$$

$$r_t \ge R_{Lt} \qquad \forall t \tag{5-8e1}$$

$$r_t \le R_{Ut} \qquad \forall t \tag{5-8e2}$$

$$-m_{ihet} + B_{ji}^{(k)}n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \tag{5-8f1}$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi}, H_{Pi}+1, \ldots, A_{Ri}-A_U-1,\ e=A_L+h, A_L+h+1, \ldots, A_U+h,\ \forall t$

$$-m_{ihet} + B_{ji}^{(k)}n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \tag{5-8f2}$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=A_{Ri}-A_U, A_{Ri}-A_U+1, \ldots, H_{Ui},\ e=A_L+h, A_L+h+1, \ldots, A_{Ri},\ \forall t$

$$m_{ihet} - B^{(k)}_{j+1,i} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \qquad (5\text{-}8f3)$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=H_{Pi},H_{Pi}+1,\ldots,A_{Ri}-A_U-1,\ e=A_L+h,A_L+h+1,\ldots,A_U+h,\ \forall t$

$$m_{ihet} - B^{(k)}_{j+1,i} n_{i,h-1,e-1,t-1} + M\delta_{ji} \le M \qquad (5\text{-}8f4)$$

$k=1,2,\ldots,\ j=1,2,\ldots,J,\ i=1,2,\ldots,I-1,\ h=A_{Ri}-A_U,A_{Ri}-A_U+1,\ldots,H_{Ui},\ e=A_L+h,A_L+h+1,\ldots,A_{Ri},\ \forall t$

$$\sum_{j=1}^{J} \delta_{ji} = 1 \qquad i=1,2,\ldots I-1 \qquad (5\text{-}8f5)$$

$$n_{ihet},\ s_t,\ r_t,\ m_{ihet} \ge 0$$

$$\delta_{ji} = 0,1$$

APPENDIX C1 - XPRESS-MP MODEL BUILDER INPUT FOR THE MIP MODEL (3-18)

```
MODEL MANPOWER PLANNING
LET II=6
LET TT=10
LET JJ=3
LET KK=6
LET r=0.1
LET M=4000
VARIABLES
  NN(II,TT)
  NP(II,TT)
  NT(TT)
  NR(1,TT)
  X1(JJ,II)
  NSKC(TT)
  NREC(TT)
  NPEC(TT)
!  Initial data and parameters
TABLES
  NI(II)
  RW(II,TT)
  RUG(II,TT)
  RLG(II,TT)
  ND(TT)
  NG(II,TT)
  RLD(TT)
  RUD(TT)
  RG(II,TT)
  NLR(1,TT)
  NUR(1,TT)
  CN(II,TT)
  CR(II,TT)
  CP(II,TT)
  ISTKC
  ISEPC
  B(KK,II)
DATA
! NI(1)=1500
! NI(2)=2000
! NI(3)=2800
! NI(4)=2100
! NI(5)=1100
! NI(6)=500
  RW(1,1)=0.01, 0.01, 0.02, 0.07, 0.06, 0.02, 0.01, 0.03, 0.02, 0.02
  RW(2,1)=0.02, 0.02, 0.03, 0.04, 0.06, 0.05, 0.03, 0.04, 0.05, 0.03
  RW(3,1)=0.10, 0.15, 0.09, 0.08, 0.07, 0.06, 0.10, 0.09, 0.10, 0.12
  RW(4,1)=0.15, 0.12, 0.10, 0.09, 0.10, 0.11, 0.13, 0.10, 0.10, 0.13
  RW(5,1)=0.10, 0.15, 0.20, 0.20, 0.10, 0.12, 0.10, 0.20, 0.15, 0.17
  RW(6,1)=0.20, 0.25, 0.15, 0.10, 0.15, 0.12, 0.13, 0.15, 0.14, 0.17
!  unit=£1000/year
  CN(1,1)=10, 11, 12, 15, 17, 20, 23, 25, 30, 31
  CN(2,1)=12, 14, 16, 19, 22, 25, 28, 30, 35, 36
  CN(3,1)=14, 16, 18, 21, 24, 27, 31, 34, 38, 39
```

```
  CN(4,1)=17, 20, 22, 25, 27, 30, 35, 38, 40, 43
  CN(5,1)=20, 23, 26, 27, 30, 33, 36, 39, 41, 44
  CN(6,1)=24, 27, 31, 32, 33, 35, 38, 41, 42, 45
  CR(1,1)=20, 22, 24, 28, 32, 36, 42, 48, 52, 58
DISKDATA
  NI(1) = MPNI
  B(1,1)= MPBOUND1.DAT
ASSIGN
  RLG(i=1:II, t=1:TT)=0.1
  RUG(i=1:II, t=1:TT)=0.05
  ND(t=1:TT)=10000
  RLD(t=1:TT)=0.1
  RUD(t=1:TT)=0.05
  RG(1, t=1:TT)=0.2
  RG(2, t=1:TT)=0.2
  RG(3, t=1:TT)=0.25
  RG(4, t=1:TT)=0.2
  RG(5, t=1:TT)=0.1
  RG(6, t=1:TT)=0.05
  NG(i=1:II, t=1:TT)=RG(i,t)*ND(t)
  NLR(1, t=1:TT)=800
  NUR(1, t=1:TT)=900
  CP(1, t=1:TT)=2.4*CN(1,t)
  CP(2, t=1:TT)=2.7*CN(2,t)
  CP(3, t=1:TT)=3.6*CN(3,t)
  CP(4, t=1:TT)=6*CN(4,t)
  CP(5, t=1:TT)=12*CN(5,t)
  CP(6, t=1:TT)=18*CN(6,t)
!   The constant value of the objective function
  ISTKC = SUM(i=1:II) (CN(i,1)/(1+r))*0.5*NI(i)
  ISEPC = SUM(i=1:II) (CP(i,1)/(1+r))*RW(i,1)*NI(i)
CONSTRAINTS
!   Objective function
!
! MPCOST: SUM(i=1:II, t=2:TT) ((CN(i,t)/(1+r)^t)*0.5)*NN(i,t)                    &
!              + SUM(i=1:II, t=2:TT) ((CN(i,t)/(1+r)^t)*0.5)*NN(i,t-1)            &
!              +SUM(i=1:II) ((CN(i,1)/(1+r))*0.5)*NN(i,1)                          &
!        +SUM(i=1:1, t=1:TT) (CR(i,t)/(1+r)^t)*NR(i,t)                             &
!        +SUM(i=1:II, t=2:TT) ((CP(i,t)/(1+r)^t)*RW(i,t))*NN(i,t-1)                &
!              $ ISTKC+ISEPC
  NMOD3: SUM(i=1:II) ( (CN(i,1)/(1+r)*0.5) + (CN(i,2)/(1+r)^2*0.5) +              &
                    (CP(i,2)/(1+r)^2*RW(i,2)) ) * NN(i,1)                          &
      +SUM(i=1:II, t=2:TT-1) ( (CN(i,t)/(1+r)^t*0.5) + (CN(i,t+1)*0.5             &
          /(1+r)^(t+1)) + (CP(i,t+1)*RW(i,t+1)/(1+r)^(t+1)) ) * NN(i,t)            &
      +SUM(i=1:II) (CN(i,TT)/(1+r)^TT*0.5) * NN(i,TT)                              &
      +SUM(i=1:1, t=1:TT) (CR(i,t)/(1+r)^t) * NR(i,t)                              &
               $ ISTKC+ISEPC
! Present different kinds of cost separately
  MSC:         NSKC(1) - SUM(i=1:II) ( CN(i,1)/(1+r)*0.5 ) * NN(i,1)               &
                         =   ISTKC
  MSC1(t=2:TT): NSKC(t) - SUM(i=1:II) ((CN(i,t)/(1+r)^t)*0.5)*NN(i,t)              &
                - SUM(i=1:II) ((CN(i,t)/(1+r)^t)*0.5)*NN(i,t-1)   =  0
  MRC(t=1:TT): NREC(t) - (CR(1,t)/(1+r)^t)*NR(1,t)                =  0
  MCP:         NPEC(1)  =  ISEPC
  MCP1(t=2:TT): NPEC(t) - SUM(i=1:II) ((CP(i,t)/(1+r)^t)*RW(i,t))*NN(i,t-1) = 0
!   Stocks
  SK11(i=1:1,t=1:1): NN(i,t)+NP(i,t)-NR(i,t) = NI(i)-NI(i)*RW(i,t)
```

```
  SKI1(i=2:II-1,t=1:1): NN(i,t)+NP(i,t)-NP(i-1,t)                        &
                                   = NI(i)-NI(i)*RW(i,t)
  SK61(i=6:6,t=1:1): NN(i,t)-NP(i-1,t) = NI(i)-NI(i)*RW(i,t)
  SKIT(i=2:II-1, t=2:TT): NN(i,t)-(1-RW(i,t))*NN(i,t-1)+NP(i,t)          &
                          -NP(i-1,t) = 0
  SK6T(i=6:6,t=2:TT): NN(i,t)-(1-RW(i,t))*NN(i,t-1)-NP(i-1,t) = 0
  SK1T(i=1:1,t=2:TT): NN(i,t)-(1-RW(i,t))*NN(i,t-1)+NP(i,t)-NR(i,t) = 0
!   Stable promotion rate
  PR1U(i=1:II-1,j=1:JJ,t=2:TT): NP(i,t)-B(2*j,i)*NN(i,t-1)+M*X1(j,i) < M
  PR1L(i=1:II-1,j=1:JJ,t=2:TT): NP(i,t)-B(2*j-1,i)*NN(i,t-1)-M*X1(j,i) > -M
  PR1U(i=1:II-1,j=1:JJ,t=1:1): NP(i,t)+M*X1(j,i) < B(2*j,i)*NI(i)+M
  PR1L(i=1:II-1,j=1:JJ,t=1:1): NP(i,t)-M*X1(j,i) > B(2*j-1,i)*NI(i)-M
!   Total force
  TFORCE(t=1:TT): NT(t)- SUM(i=1:II) NN(i,t) = 0
  UFORCE(t=1:TT): NT(t) < (1+RUD(t))*ND(t)
  LFORCE(t=1:TT): NT(t) > (1-RLD(t))*ND(t)
!   The number of the people for each rank
  UPEOPLE(i=1:II, t=1:TT): NN(i,t) < (1+RUG(i,t))*NG(i,t)
  LPEOPLE(i=1:II, t=1:TT): NN(i,t) > (1-RLG(i,t))*NG(i,t)
!   The number of recruitment
  UACCE(i=1:1, t=1:TT): NR(i,t) < NUR(i,t)
  LACCE(i=1:1, t=1:TT): NR(i,t) > NLR(i,t)
!   Binary variables
  BINARYV(i=1:II-1): SUM(j=1:JJ) X1(j,i) = 1
BOUNDS
  X1(j=1:JJ, i=1:II-1) $ 1
GENERATE MOD3-18.MAT
```

APPENDIX C2 - XPRESS-MP MODEL BUILDER INPUT FOR THE MIP MODEL (5-9)

```
MODEL MANPOWER PLANNING--2 RANGES
! A model involing Grade and TLS (Total Length of Service).
LET II=6          ! the number of grades
LET TT=10         ! the number of planning years
LET JJ=2          ! the number of ranges for promotion rate
LET KK=4          ! the number of bound values, each range gets 2 bounds
LET M=100000      ! big M value
LET CRT= 20       ! average recruitment cost per person in initial year: £K/year
LET SI= 10        ! initial salary, i.e., the salary for recruits: £K/year
LET SA= 0.5       ! annual salary increment in terms of TLS: £K/year
LET r=0.05        ! annual discount rate
LET s=0.06        ! annual rate of increase of salary
LET AMO= 23       ! the mode of the recruitment ages
!  Since the minimum subscript in the XPRESS-MP software should be more than 1
!  the initial value of TLS, i.e., subscript h, is 1 rather than 0.
!  Therefore, appropriate ages must be increased 1 year
LET AX= 85+1      ! maximum life expectancy
LET LLP= 20+1     ! required minimum TLS for annual pension entitlement
!  Initial data and parameters
TABLES
  LRS(II-1)       ! required minimum TLS for promotion from grade i to i+1
  LMI(II)         ! minimum TLS of staff in grade i, LMI(i)=LRS(i-1), LMI(1)=1
  LMX(II)         ! maximum TLS in grade i
  AR(II)          ! retirement age in each grade i
! SP(II-1)        ! separation point in terms of TLS for distinguishing
                  !  promotion zones
DATA
  LRS(1)=2, 4, 8, 12, 15          ! each data has been increased 1 year
  LMI(1)=1, 2, 4, 8, 12, 15       ! each data has been increased 1 year
  AR(1)=26, 31, 37, 45, 49, 52    ! each data has been increased 1 year
! SP(1)=2, 7, 12, 21, 22          ! each data has been increased 1 year
ASSIGN
  LMX(i=1:II)= AR(i)-AMO
  HH=AR(II)-AMO         ! maximum TLS in the system
TABLES
  NI(II,HH)             ! number of staff in grade i with total length of service h
                        !  initially, i.e., at the end of year 0
  RT(II,HH,TT)          ! wastage rate in grade i with total length of service h
                        !  at end of year t
  RTX(II,HH,TT)         ! wastage rate for whatever reason except death of individuals
                        !  in grade i with total length of service h at end of year t
  ND(TT)                ! target total number of staff at end of year t
  RUD(TT)               ! maximum upper proportional deviation in target total
                        !  number of staff at end of year t
  RLD(TT)               ! maximum lower proportional deviation in target total
                        !  number of staff at end of year t
  RUG(II,TT)            ! maximum upper proportional deviation in target number
                        !  of staff in grade i at end of year t
  RLG(II,TT)            ! maximum lower proportional deviation in target number
                        !  of staff in grade i at end of year t
  RG(II,TT)             ! target proportion of staff in grade i at end of year t
```

```
  NUR(TT)             ! upper bound on the number of recruits to grade 1
                      !  at end of year t
  NLR(TT)             ! lower bound on the number of recruits to grade 1
                      !  at end of year t
  B1(KK,II-1)         ! bound values for each promotion range within promotion
                      !  zone 1
! B2(KK,II-1)         ! bound values for each promotion range within promotion
                      !  zone 2
  SR(II)              ! salary rate of grade i as a times of salary for
                      !  recruits, SR(1)=1
  CR(TT)              ! average recruitment cost per person in year t
  CN(II,HH,TT)        ! average annual salary per person in grade i with total
                      !  length of service h in year t
  CLS(II,HH,TT)       ! average lump sum payment per person for those who leave
                      !  the system in grade i with total length of service h
                      !  in year t
  CLP(II,HH,TT)       ! average annual pension per person for those who
                      !  leave the system in grade i with total length of
                      !  service h in year t
  PS(AX,AX+1)         ! the probability of survival from age e to age e+y
  PB(AX)              ! the probability of survival from age e to age e+1
  RT1(II,LMX(II))     ! wastage rates which only involve grade and TLS
  BV(II,HH)           ! binary coefficients for forcing variables NP(i-1,h,e,t)
                      ! to be zero as h execesses LMX(i-1)
! wastage occurs only after minimum total length of service, LMI(i)
DATA
  RT1(1,LMI(1)+1)= .0031, .0766
  RT1(2,LMI(2)+1)= .1029, .1157, .1365, .1012, .1263, .1503
  RT1(3,LMI(3)+1)= .0781, .0624, .0936, .0742, .0824, .0504, .3014, .0899, .1223, .1650
  RT1(4,LMI(4)+1)= .0265, .0377, .1820, .0840, .1847, .1192, .0741, .1502, .0659, .0778, .0791, .0309, .5137, .5439
  RT1(5,LMI(5)+1)= .1341, .1004, .0866, .1625, .1021, .0823, .0816, .0400, .3676, .2261, .2932, .2500, .2768, .5065
  RT1(6,LMI(6)+1)= .0612, .0240, .0470, .0320, .0295, .1741, .1631, .1537, .2393, .2667, .2010, .2961, .2812, .5983
  SR(1)= 1, 1.1, 1.3, 1.5, 1.8, 2
ASSIGN
  RT(i=1:II,h=LMI(i)+1:LMX(i),t=1:TT)=RT1(i,h) ! the same grade and TLS
                                               ! in any year gets the same wastage rate
  RLG(i=1:II, t=1:TT)=0.15
  RUG(i=1:II, t=1:TT)=0.15
  ND(t=1:TT)=30000
  RLD(t=1:TT)=0.15
  RUD(t=1:TT)=0.15
  RG(1, t=1:TT)=0.09
  RG(2, t=1:TT)=0.27
  RG(3, t=1:TT)=0.25
  RG(4, t=1:TT)=0.20
  RG(5, t=1:TT)=0.13
  RG(6, t=1:TT)=0.06
  NLR(t=1:TT)=2000
  NUR(t=1:TT)=3000
  BV(i=1:II-1,h=1:HH ¦ h .gt. LMX(i) )=0
  BV(i=1:II-1,h=1:HH ¦ h .le. LMX(i) )=1
!  initial value of h is 1 rather than 0
  CR(t=1:TT)= CRT*(1+s)^t
  CN(i=1:II,h=1:HH,t=1:TT)= (SR(i)*SI+(h-1)*SA)*(1+s)^t
  CLS(i=1:II,h=1:HH,t=1:TT)= 0.4*(h-1)*CN(i,h,t)
  CLP(i=1:II,h=1:HH,t=1:TT)= 0.8*CN(i,h,t)
```

```
TABLES
!  The number of staff in each grade initially, i.e. at end of year 0, does
!  not include the number of staff in maximum TLS, i.e. LMX(i), since the
!  number of staff in LMX(i) represents the number of recruitment.
  NI1(LMX(1)-1)
  NI2(LMX(2)-1)
  NI3(LMX(3)-1)
  NI4(LMX(4)-1)
  NI5(LMX(5)-1)
  NI6(LMX(6)-1)
DISKDATA
!  disk files for the number of staff in each grade at end of year 0
  NI1(LMI(1))=NI1.DAT
  NI2(LMI(2))=NI2.DAT
  NI3(LMI(3))=NI3.DAT
  NI4(LMI(4))=NI4.DAT
  NI5(LMI(5))=NI5.DAT
  NI6(LMI(6))=NI6.DAT
  B1(1,1)= MPBOUND1.DAT  ! the bound values of possible ranges for promotion zone 1
! B2(1,1)=MPBOUND2.DAT   ! the bound values of possible ranges for promotion zone 2
  PB(23+1)= PB.DAT       ! the probabilities in PB.DAT file begin from
          ! age 23 to 85. Since initial TLS is 1, each age in the PB.DAT file
          ! should be shifted 1 year, i.e., the probability of age e in the
          ! file is equivalent to the probability of age e+1 in the program
ASSIGN
  PS(e=AMO+1:AX,e+1)=PB(e)      ! note that AX has been increased 1 year
  PS(e=AMO+1:AR(II),e)=1
  PS(e=AMO+1:AR(II),y=e+1:AX)=PS(e,y-1)*PS(y-1,y)       ! note that AX has been
                                                        !  increased 1 year
  RTX(i=1:II,h=LMI(i)+1:LMX(i),t=1:TT)=RT(i,h,t)-(1-PB(AMO+h))
  NI(1,h=LMI(1):LMX(1)-1)=NI1(h)
  NI(2,h=LMI(2):LMX(2)-1)=NI2(h)
  NI(3,h=LMI(3):LMX(3)-1)=NI3(h)
  NI(4,h=LMI(4):LMX(4)-1)=NI4(h)
  NI(5,h=LMI(5):LMX(5)-1)=NI5(h)
  NI(6,h=LMI(6):LMX(6)-1)=NI6(h)
!   The constant value of the objective function
!     The constant amount of the lump sum payment
  CNSCLUM=  SUM(i=1:II,h=LMI(i)+1:LLP-1)                              &
              CLS(i,h,1)/(1+r)*RTX(i,h,1)*NI(i,h-1)
!     The constant amount of the annual pension
  CNSCLIF=  SUM(i=4:II,h=LLP:LMX(i),y=1:AX-h-AMO)                     &
        CLP(i,h,1)*(1+s)^y/(1+r)^(1+y)*PS(h+AMO,h+AMO+y)*RTX(i,h,1)*NI(i,h-1)
!     The coeficients of the annual pension
TABLES
  CA(II,TT)
  CB(II,HH,TT)
ASSIGN
  CA(i=4:II,t=1:TT)=SUM(y=1:AX-AR(i))                                  &
           CLP(i,LMX(i),t)*(1+s)^y/(1+r)^(t+y)*PS(AR(i),AR(i)+y)
  CB(i=4:II,h=LLP:LMX(i),t=2:TT)=SUM(y=1:AX-h-AMO)                     &
           CLP(i,h,t)*(1+s)^y/(1+r)^(t+y)*PS(h+AMO,h+AMO+y)*RTX(i,h,t)
VARIABLES
  N(6,30,10)           ! number of staff in grade i with
                       ! total length of service h at end of year t
  NT(t=1:TT)           ! the total number of staff in the system at end of year t
  NR(t=1:TT)           ! number of recruits to grade 1 at end of year t
```

```
  NP(6,30,10)          ! number of staff promoted from grade i to i+1
                       ! with total length of service h at end of year t
  X1(JJ,II-1)          ! binary variables for promotion zone 1
! X2(JJ,II-1)          ! binary variables for promotion zone 2
  NREC(t=1:TT)         ! recruitment costs of the system in year t
  NSKC(t=1:TT)         ! stock costs of the system in year t
  NLMC(t=1:TT)         ! lump sum payment of the system in year t
  NLFC(t=1:TT)         ! annual pension of the system in year t
CONSTRAINTS
!   Objective function
  NPCOST:  SUM(t=1:TT) NREC(t)+SUM(t=1:TT) NSKC(t)+SUM(t=1:TT) NLMC(t)+      &
           SUM(t=1:TT) NLFC(t)  $
!  Present different kinds of costs separately
!     recruitment costs
  MRC(t=1:TT): NREC(t) - (CR(t)/(1+r)^t)*NR(t) = 0
!     stock costs
  MSC(t=1:TT): NSKC(t)-SUM(i=1:II,h=LMI(i):LMX(i)-1) CN(i,h,t)/(1+r)^t*N(i,h,t)=0
!     lump sum payment
  MLUMC1:          NLMC(1) = CNSCLUM
  MLUMCS(t=2:TT): NLMC(t) - SUM(i=1:II,h=LMI(i)+1:LLP-1)                     &
                  CLS(i,h,t)/(1+r)^t*RTX(i,h,t)*N(i,h-1,t-1) = 0
!     annual pension
! MLIFC1:          NLFC(1) - SUM(i=4:II,y=1:AX-AR(i))                        &
!                  CLP(i,LMX(i),1)*(1+s)^y/(1+r)^(1+y)*PS(AR(i),AR(i)+y)*    &
!                  N(i,LMX(i),1) = CNSCLIF
! MLIFCS(t=2:TT): NLFC(t)-                                                   &
!       SUM(i=4:II,h=LLP:LMX(i),y=1:AX-h-AMO)                                &
!          CLP(i,h,t)*(1+s)^y/(1+r)^(t+y)*PS(h+AMO,h+AMO+y)*                 &
!          RTX(i,h,t)*N(i,h-1,t-1)-                                          &
!       SUM(i=4:II,y=1:AX-AR(i))                                             &
!        CLP(i,LMX(i),t)*(1+s)^y/(1+r)^(t+y)*PS(AR(i),AR(i)+y)*N(i,LMX(i),t)=0
  MLIFC1:          NLFC(1)-SUM(i=4:II) CA(i,1)*N(i,LMX(i),1) = CNSCLIF
  MLIFCS(t=2:TT): NLFC(t)-SUM(i=4:II,h=LLP:LMX(i)) CB(i,h,t)*N(i,h-1,t-1)    &
                  -SUM(i=4:II) CA(i,t)*N(i,LMX(i),t)=0
!  Stocks
  BB1(h=LMI(1):LMI(1),t=1:TT): N(1,h,t)-NR(t)=0
  B3A(h=LRS(1):LMX(1)): N(1,h,1)+NP(1,h,1)=(1-RT(1,h,1))*NI(1,h-1)
  B3(h=LRS(1):LMX(1),t=2:TT): N(1,h,t)+NP(1,h,t)-(1-RT(1,h,t))*N(1,h-1,t-1)=0
  B4A(h=LMI(II)+1:LMX(II)): N(II,h,1)-BV(II-1,h)*NP(II-1,h,1)=               &
        (1-RT(II,h,1))*NI(II,h-1)
  B4(h=LMI(II)+1:LMX(II),t=2:TT): N(II,h,t)-                                 &
        BV(II-1,h)*NP(II-1,h,t)-(1-RT(II,h,t))*N(II,h-1,t-1)=0
  B5(i=2:II,h=LMI(i):LMI(i),t=1:TT): N(i,h,t)-BV(i-1,h)*NP(i-1,h,t)=0
  B6A(i=2:II-1,h=LMI(i)+1:LRS(i)-1): N(i,h,1)-BV(i-1,h)*NP(i-1,h,1)=         &
        (1-RT(i,h,1))*NI(i,h-1)
  B6(i=2:II-1,h=LMI(i)+1:LRS(i)-1,t=2:TT): N(i,h,t)-                         &
        BV(i-1,h)*NP(i-1,h,t)-(1-RT(i,h,t))*N(i,h-1,t-1)=0
  B7A(i=2:II-1,h=LRS(i):LMX(i)): N(i,h,1)-BV(i-1,h)*NP(i-1,h,1)+             &
        NP(i,h,1)=(1-RT(i,h,1))*NI(i,h-1)
  B7(i=2:II-1,h=LRS(i):LMX(i),t=2:TT): N(i,h,t)-                             &
        BV(i-1,h)*NP(i-1,h,t)+NP(i,h,t)-(1-RT(i,h,t))*N(i,h-1,t-1)=0
!  Total number of staff
  C1(t=1:TT): NT(t)-SUM(i=1:II,h=LMI(i):LMX(i)-1) N(i,h,t)=0
  C2(t=1:TT): -NT(t) < -(1-RLD(t))*ND(t)
  C3(t=1:TT):  NT(t) < (1+RUD(t))*ND(t)
!   The number of the staff for each grade
```

```
 D1(i=1:1 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 D2(i=1:1 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
 D3(i=2:2 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 D4(i=2:2 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
 D5(i=3:3 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 D6(i=3:3 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
 D7(i=4:4 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 D8(i=4:4 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
 D9(i=5:5 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 DX(i=5:5 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
 DA(i=6:6 , t=1:TT): -SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                    &
        < -RG(i,t)*ND(t)*(1-RLG(i,t))
 DB(i=6:6 , t=1:TT): SUM(h=LMI(i):LMX(i)-1) N(i,h,t)                     &
        <  RG(i,t)*ND(t)*(1+RUG(i,t))
!  The number of recruitment
 E1(t=1:TT): -NR(t) < -NLR(t)
 E2(t=1:TT):  NR(t) <  NUR(t)
!  Stable promotion rates
 FA(i=1:II-1,h=LRS(i):LMX(i),j=1:JJ): -NP(i,h,1)                         &
       +M*X1(j,i) < -B1(2*j-1,i)*NI(i,h-1)+M
 F(i=1:II-1,h=LRS(i):LMX(i),j=1:JJ,t=2:TT): -NP(i,h,t)                   &
       +B1(2*j-1,i)*N(i,h-1,t-1)+M*X1(j,i) <  M
 GA(i=1:II-1,h=LRS(i):LMX(i),j=1:JJ): NP(i,h,1)                          &
       +M*X1(j,i) < B1(2*j,i)*NI(i,h-1)+M
 G(i=1:II-1,h=LRS(i):LMX(i),j=1:JJ,t=2:TT):  NP(i,h,t)                   &
       -B1(2*j,i)*N(i,h-1,t-1)+M*X1(j,i) <  M
! FA(i=1:II-1,h=LRS(i):SP(i),j=1:JJ): -NP(i,h,1)                         &
!      +M*X1(j,i) < -B1(2*j-1,i)*NI(i,h-1)+M
! F(i=1:II-1,h=LRS(i):SP(i),j=1:JJ,t=2:TT): -NP(i,h,t)                   &
!      +B1(2*j-1,i)*N(i,h-1,t-1)+M*X1(j,i) <  M
! GA(i=1:II-1,h=LRS(i):SP(i),j=1:JJ): NP(i,h,1)                          &
!      +M*X1(j,i) < B1(2*j,i)*NI(i,h-1)+M
! G(i=1:II-1,h=LRS(i):SP(i),j=1:JJ,t=2:TT):  NP(i,h,t)                   &
!      -B1(2*j,i)*N(i,h-1,t-1)+M*X1(j,i) <  M
! HA(i=1:II-1,h=SP(i)+1:LMX(i),j=1:JJ): -NP(i,h,1)                       &
!      +M*X2(j,i) < -B2(2*j-1,i)*NI(i,h-1)+M
! H(i=1:II-1,h=SP(i)+1:LMX(i),j=1:JJ,t=2:TT): -NP(i,h,t)                 &
!      +B2(2*j-1,i)*N(i,h-1,t-1)+M*X2(j,i) <  M
! IA(i=1:II-1,h=SP(i)+1:LMX(i),j=1:JJ): NP(i,h,1)                        &
!      +M*X2(j,i) < B2(2*j,i)*NI(i,h-1)+M
! I(i=1:II-1,h=SP(i)+1:LMX(i),j=1:JJ,t=2:TT):  NP(i,h,t)                 &
!      -B2(2*j,i)*N(i,h-1,t-1)+M*X2(j,i) <  M
! The promotion rate in promotion zone 1 is greater than or
! equal to that in promotion zone 2
! PRZN(i=1:II-1): -SUM(j=1:JJ) B1(2*j-1,i)*X1(j,i)+SUM(j=1:JJ)           &
!                     B2(2*j-1,i)*X2(j,i) < 0
```

```
!   Binary variables
  BV1A(i=1:II-1): SUM(j=1:JJ) X1(j,i) =1
! BV2B(i=1:II-1): SUM(j=1:JJ) X2(j,i) =1
BOUNDS
  X1(j=1:JJ, i=1:II-1) $ 1
! X2(j=1:JJ, i=1:II-1) $ 1
GENERATE MOD5-9.MAT
```

APPENDIX C3 - PROGRAM FOR GENERATING BOUNDS OF PROMOTION RATES, OVERLAPPING RANGES AND RESULTS OF THE MODELS

```
250 CLS
300 DIM N(6, 31, 10), NA(6, 10), NB(31, 10), NT(10)
350 DIM NNA(10, 10), NNB(10, 31), NNC(10)
400 DIM NP(6, 31, 10), NPA(10, 10), NPB(10, 31), NPC(10)
450 DIM NR(10), LRS(6), LMI(6), LMX(6), AR(6)
500 DIM RP(6, 31, 10), RPA(6, 10), RPB(6, 31), RPC(6), PTTLS(31, 10), PTGRD(6, 10)
550 DIM NWGA(6, 31, 10), NWGB(6, 10), NWGC(31, 10), NWGT(10), RIW(6, 31)
570 DIM NRT(10, 10), NRTA(10), NRTB(10)
600 DIM REC(10), SKC(10), LMC(10), LFC(10), TCOST(10)
610 DIM B(3, 20, 6), BN(3, 20, 6), BL(3, 6), BU(3, 6), X(3, 20, 6)
620 DIM BUS(3, 6), BLS(3, 6), BUT(3, 6)
630 DIM MPN(6, 10), MPNP(6, 10), MPNT(10), MPNR(6, 10), MPSC(10), MPRC(10), MPPC(10)
650 DIM BL1(10), BU1(10), RP1(6, 10)
800 REM ************************************************************
850 REM *     FIRST SCREEN                                           *
900 REM ************************************************************
950 LOCATE 2, 23: PRINT " University of Edinburgh "
1000 LOCATE 3, 23: PRINT "    Cheng-Liang Yang "
1050 LOCATE 4, 1: FOR J = 1 TO 79: PRINT "="; : NEXT J
1100 LOCATE 7, 5: PRINT "1. NARROW BOUNDS OF PROMOTION RATES           :"
1150 LOCATE 11, 5: PRINT "2. SHIFT BOUNDS OF PROMOTION RATES           :"
1200 LOCATE 15, 5: PRINT "3. WRITE RESULTS                             :"
1250 LOCATE 17, 5: PRINT "SELECT 1, 2, OR 3, PLEASE                    :"
1400 LOCATE 17, 55: INPUT Y$
1450 WHILE VAL(Y$) = 1
1500   LOCATE 9, 5: PRINT " enter reduction factor, Q                  :"
1600   LOCATE 9, 55: INPUT Q
1700   GOTO 2650
1750 WEND
1800 WHILE VAL(Y$) = 2
1850   LOCATE 12, 5: PRINT "  enter extension factor, Q*               :"
1900   LOCATE 13, 5: PRINT "  enter reduction factor, Q                :"
1950   LOCATE 14, 5: PRINT "  enter the number of promotion ranges, J*  :"
2000   LOCATE 12, 55: INPUT Q1
2050   LOCATE 13, 55: INPUT Q
2100   LOCATE 14, 55: INPUT N
2150   GOTO 3510
2200 WEND
2210 WHILE VAL(Y$) = 3
2220   CLS
2230   LOCATE 3, 7: PRINT "IT IS WORKING, BE PATIENT PLEASE"
2300   GOTO 3600
2350 WEND
2400 LOCATE 17, 55: PRINT SPACE$(15): GOTO 1400
2450 REM ****************************************************************
2500 REM *  1. NARROW BOUNDS OF PROMOTION RATE                           *
2600 REM ****************************************************************
2650 CLS
2700 GOSUB 11150     'retrive binary values from MIPX.ASC file
```

```
2750 REM     ******************************************
2800 REM     *  copy MIPOUT.ASC to MIPOUT.DAT file    *
2900 REM     ******************************************
2950 OPEN "MIPOUT.ASC" FOR INPUT AS £6
3000 OPEN "MIPOUT.DAT" FOR OUTPUT AS £7
3050 LINE INPUT £6, OUTVAL$
3100 PRINT £7, OUTVAL$
3150 IF NOT EOF(6) THEN 3050
3200 CLOSE £6, £7
3250 FINAME$ = "MPBOUND"
3300 GOSUB 4850       'read bound values from MPBOUND*.DAT files
3310 GOSUB 5900       'copy MPBOUND*.DAT to OLDBOUND*.DAT
3320 GOSUB 6650       'calculate new bounds and write them into MPBOUND*.DAT
3350 SYSTEM
3400 REM ****************************************************************
3410 REM *  2. SHIFT BOUNDS OF PROMOTION RATE                           *
3420 REM ****************************************************************
3510 GOSUB 4000       'read the values of binary variables from MIPX.DAT
3520 GOSUB 4850
3530 GOSUB 6650
3540 SYSTEM
3550 REM ****************************************************************
3560 REM *  3. PRINT OUTPUT                                             *
3570 REM ****************************************************************
3600 OPEN "MIPOUT.DAT" FOR INPUT AS £1
3610 LINE INPUT £1, OUTVAL$
3620 COST = VAL(MID$(OUTVAL$, 28, 12))
3630 WHILE MID$(OUTVAL$, 9, 8) = "NMOD3   "
3640 REM     ******************************************
3645 REM     *  print output for model 3-18           *
3650 REM     ******************************************
3660   GOSUB 40000
3670   GOSUB 4000
3680   GOSUB 4850
3690   GOSUB 40815
3700   SYSTEM
3710 WEND
3720 REM     ******************************************
3730 REM     *  print output for model 5-9            *
3740 REM     ******************************************
3750 GOSUB 15900
3760 SYSTEM
3950 REM ****************************************************************
3960 REM *  SUBROUTINE:                                                 *
3970 REM *  READ THE VALUES OF BINARY VARIABLES FROM MIPX.DAT FILE      *
3980 REM ****************************************************************
4000 CLS
4010 OPEN "MIPX.DAT" FOR INPUT AS £3  'THE VALUES OF BINARY VARIABLES, X(Z,J,I)
4020 INPUT £3, NPRZON, JJ, II  'the number of promotion zones, ranges,
4030 FOR Z = 1 TO NPRZON        ' and grades
4040   COUNT = 0
4050   FOR J = 1 TO JJ
4060     FOR I = 1 TO II - 1
4070       WHILE EOF(3)
4080         GOSUB 10150: SYSTEM
4090       WEND
4100       INPUT £3, X(Z, J, I)
```

```
4110        IF X(Z, J, I) = 1 THEN COUNT = COUNT + 1
4150      NEXT I
4200    NEXT J
4250 IF COUNT <> II - 1 THEN CLS :PRINT:PRINT:PRINT TAB(10);"INFEASIBLE SOLUTION !":SYSTEM
4300    IF Z = NPRZON THEN 4450
4350    LINE INPUT £3, STAR$  'stars line for separating promotion zones
4400 NEXT Z
4450  WHILE NOT EOF(3)
4500    GOSUB 10150: SYSTEM
4550  WEND
4600 CLOSE £3
4650 FINAME$ = "OLDBOND"
4660 RETURN
4700 REM***************************************************************
4750 REM*  READ BOUND VALUES FROM DISK FILES, MPBOUND*.DAT OR OLDBOND*.DAT  *
4800 REM***************************************************************
4850 FOR Z = 1 TO NPRZON
4900   Z1$ = STR$(Z): Z2$ = RIGHT$(Z1$, LEN(Z1$) - 1)
4950   FI$ = FINAME$ + Z2$ + ".DAT"
5000   OPEN FI$ FOR INPUT AS £3
5050     FOR J = 1 TO 2 * JJ
5100       FOR I = 1 TO II - 1
5150         WHILE EOF(3)
5200           GOSUB 10650: SYSTEM  'the number of promotion ranges and grades
5250         WEND  'contradictory to those in the MPBOUND*.DAT or OLDBOND*.DAT
5300         INPUT £3, B(Z, J, I)
5350       NEXT I
5400     NEXT J
5450   WHILE NOT EOF(3)
5500     GOSUB 10650: SYSTEM
5550   WEND
5600 CLOSE £3
5650 NEXT Z
5660 RETURN
5750 REM **************************************************************
5800 REM *  COPY MPBOUND*.DAT TO OLDBOND*.DAT                            *
5850 REM **************************************************************
5900 FOR Z = 1 TO NPRZON
5950   Z1$ = STR$(Z): Z2$ = RIGHT$(Z1$, LEN(Z1$) - 1)
6000   FI$ = "OLDBOND" + Z2$ + ".DAT"
6050   OPEN FI$ FOR OUTPUT AS £4
6100   FOR J = 1 TO 2 * JJ
6150     FOR I = 1 TO II - 1
6200       IF I = II - 1 THEN PRINT £4, USING "£.££££"; B(Z, J, I): GOTO 6300
6250       PRINT £4, USING "£.££££"; B(Z, J, I); : PRINT £4, ",       ";
6300     NEXT I
6350   NEXT J
6400 CLOSE £4
6450 NEXT Z
6460 RETURN
6500 REM **************************************************************
6550 REM *  CALCULATE THE NEW BOUND VALUES FOR EACH GRADE               *
6600 REM **************************************************************
6650 FOR Z = 1 TO NPRZON
6700   FOR I = 1 TO II - 1
6750     FOR J = 1 TO JJ
6800       IF X(Z, J, I) = 1 THEN GOSUB 8050  'CACULATE THE NEW BOUND VALUES
```

```
6850      NEXT J
6900    NEXT I
6950 NEXT Z
7000 REM ****************************************************************
7050 REM *   WRITE THE NEW BOUND VALUES INTO THE MPBOUND*.DAT FILES        *
7100 REM ****************************************************************
7150 IF VAL(Y$) = 2 THEN JJ = N
7200 FOR Z = 1 TO NPRZON
7250   Z1$ = STR$(Z): Z2$ = RIGHT$(Z1$, LEN(Z1$) - 1)
7300   FI$ = "MPBOUND" + Z2$ + ".DAT"
7350   OPEN FI$ FOR OUTPUT AS £5  'the new bound values, BN(Z,JJ,I)
7400   FOR K = 1 TO 2 * JJ
7450     FOR I = 1 TO II - 1
7500       IF I = II - 1 THEN PRINT £5, USING "£.££££"; BN(Z, K, I): GOTO 7600
7550       PRINT £5, USING "£.££££"; BN(Z, K, I); : PRINT £5, ",      ";
7600     NEXT I
7650   NEXT K
7700 CLOSE £5
7750 NEXT Z
7760 RETURN
7850 REM       ********************************************************
7900 REM       *   SUBROUTINES : CACULATING THE NEW BOUND VALUES         *
7950 REM       *                     FOR EACH GRADE                       *
8000 REM       ********************************************************
8050 BL(Z, I) = B(Z, 2 * J - 1, I)
8100 BU(Z, I) = B(Z, 2 * J, I)
8150 H = BU(Z, 1) - BL(Z, 1)'the width in all grades will follow grade 1
8200 HN = Q * H                ' for avoiding difference
8250 IF VAL(Y$) = 2 THEN GOSUB 12950: RETURN 'SHIFTING BOUND VALUES
8300 AF = (JJ * Q - 1) / 2
8350 WHILE AF < 0 OR Q >= 1
8400   LOCATE 19, 7: PRINT "ERROR ! REDUCTION FACTOR, Q, MUST BE GREATER"
8450   LOCATE 20, 7: PRINT "THAN OR EQUAL TO "
8500   LOCATE 20, 24: PRINT 1 / JJ; " AND LESS THAN 1"
8550 GOTO 950
8600 WEND
8650 C = AF * H
8700 IF BU(Z, I) + C >= 1 THEN GOSUB 8850: RETURN
8750 IF BL(Z, I) - C <= 0 THEN BN(Z, 1, I) = 0: GOSUB 9400: RETURN
8800 BN(Z, 1, I) = BL(Z, I) - C: GOSUB 9400: RETURN
8850 BN(Z, 2 * JJ, I) = 1: BN(Z, 2 * JJ - 1, I) = 1 - HN
8900 WHILE JJ > 1
8950 FOR K = 2 * JJ - 2 TO 2 STEP -2
9000 BN(Z, K, I) = BN(Z, K + 1, I)
9050 BN(Z, K - 1, I) = BN(Z, K, I) - HN
9100 IF BN(Z, K, I) < 0 THEN BN(Z, K, I) = 0
9150 IF BN(Z, K - 1, I) < 0 THEN BN(Z, K - 1, I) = 0
9200 NEXT K
9250 GOTO 9350
9300 WEND
9350 RETURN
9400 BN(Z, 2, I) = BN(Z, 1, I) + HN
9450 WHILE JJ > 1
9500 FOR K = 3 TO 2 * JJ - 1 STEP 2
9550 BN(Z, K, I) = BN(Z, K - 1, I)
9600 BN(Z, K + 1, I) = BN(Z, K, I) + HN
9650 IF BN(Z, K, I) > 1 THEN BN(Z, K, I) = 1
```

```
9700 IF BN(Z, K + 1, I) > 1 THEN BN(Z, K + 1, I) = 1
9750 NEXT K
9800 GOTO 9900
9850 WEND
9900 RETURN
9950  REM ***************************************************************
10000 REM *   SUBROUTINE : PRINT THE ERROR MESSAGE OF READING BINARY       *
10050 REM *                   VARIABLES FROM INPUT FILE, MIPX.DAT          *
10100 REM ***************************************************************
10150 CLS
10200 PRINT : PRINT : PRINT : PRINT
10250 LOCATE 5, 16: PRINT "ERROR ! THE NUMBER OF GRADES AND THE NUMBER OF "
10300 LOCATE 6, 16: PRINT "PROMOTION RANGES CONTRODICTORY TO THOSE IN "
10350 LOCATE 7, 16: PRINT "THE MIPX.DAT FILE."
10400 RETURN
10450 REM ***************************************************************
10500 REM *   SUBROUTINE : PRINT THE ERROR MESSAGE OF READING THE          *
10550 REM *  BOUND VALUES FROM MPBOUND*.DAT OR OLDBOND*.DAT FILES          *
10600 REM ***************************************************************
10650 CLS
10700 PRINT : PRINT : PRINT : PRINT
10750 LOCATE 5, 16: PRINT "ERROR ! THE NUMBER OF GRADES AND THE NUMBER OF "
10800 LOCATE 6, 16: PRINT "PROMOTION RANGES CONTRADICTORY TO THOSE IN  "
10850 LOCATE 7, 16: PRINT "THE MPBOUND*.DAT OR OLDBOND*.DAT FILES "
10900 RETURN
10950 REM ***************************************************************
11000 REM *   SUBROUTINE : RETRIVE BINARY VALUES FROM THE OUTPUT OF THE   *
11050 REM *  MIP MODEL, MIPX.ASC FILE, AND WRITE THEM INTO MIPX.DAT FILE  *
11100 REM ***************************************************************
11150 OPEN "MIPX.ASC" FOR INPUT AS £1
11200 NPRZON = 1            'the number of promotion zone
11250 II = 1                'the number of grade
11300 JJ = 1                'the number of promotion ranges
11350 LINE INPUT £1, BI$
11400 Z = VAL(MID$(BI$, 10, 1))        'promotion zone
11450 J = VAL(MID$(BI$, 13, 2))        'promotion range
11500 I = VAL(MID$(BI$, 15, 2))        'grade
11550 IF Z > NPRZON THEN NPRZON = Z
11600 IF J > JJ THEN JJ = J
11650 IF I > II THEN II = I
11700 X(Z, J, I) = VAL(MID$(BI$, 28, 12))
11750 IF NOT EOF(1) THEN 11350
11800 II = II + 1   'because the maximum number of grade in MIPX.ASC is II-1
11850 CLOSE £1
11900 GOSUB 14950   'check binary variables
11950 REM *****************************************************************
12000 REM *  WRITE THE VALUES OF BINARY VARIABLES INTO DISK FILE, MIPX.DAT  *
12050 REM *****************************************************************
12100 OPEN "MIPX.DAT" FOR OUTPUT AS £2   'THE VALUES OF BINARY VARIABLES, X(Z,J,I)
12150 WRITE £2, NPRZON, JJ, II
12200 FOR Z = 1 TO NPRZON
12250   FOR J = 1 TO JJ
12300     FOR I = 1 TO II - 1
12350       IF I = II - 1 THEN PRINT £2, X(Z, J, I): GOTO 12450
12400       PRINT £2, X(Z, J, I); : PRINT £2, ",  ";
12450     NEXT I
12500   NEXT J
```

```
12550   IF Z = NPRZON THEN 12650
12600   PRINT £2, "******"
12650 NEXT Z
12700 CLOSE £2
12750 RETURN
12800 REM ***************************************************************
12850 REM *   SUBROUTINE : SHIFTING BOUND VALUES                            *
12900 REM ***************************************************************
12950 C1 = Q1 * H
13000 BUS(Z, I) = BU(Z, I) + C1
13050 BLS(Z, I) = BL(Z, I) - C1
13100 WHILE N * HN < (BUS(Z, I) - BLS(Z, I))
13150   CLS
13200   LOCATE 19, 7: PRINT "ERROR! THERE IS NO OVERLAPPING RANGE. CHECK EXTENSION"
13250   LOCATE 20, 7: PRINT "FACTOR, REDUCTION FACTOR AND THE NUMBER OF PROMOTION"
13300   LOCATE 21, 7: PRINT "RATE AGAIN."
13350 GOTO 950
13400 WEND
13450 BUT(Z, I) = BUS(Z, I) - HN
13500 WS = (BUT(Z, I) - BLS(Z, I)) / (N - 1)
13550 IF BUS(Z, I) >= 1 THEN GOSUB 13700: RETURN
13600 IF BLS(Z, I) <= 0 THEN BN(Z, 1, I) = 0: GOSUB 14250: RETURN
13650 BN(Z, 1, I) = BLS(Z, I): GOSUB 14250: RETURN
13700 BN(Z, 2 * N, I) = 1: BN(Z, 2 * N - 1, I) = 1 - HN
13750 WHILE N > 1
13800   FOR K = 2 * N - 2 TO 2 STEP -2
13850     BN(Z, K, I) = BN(Z, K + 2, I) - WS
13900     BN(Z, K - 1, I) = BN(Z, K, I) - HN
13950     IF BN(Z, K, I) < 0 THEN BN(Z, K, I) = 0
14000     IF BN(Z, K - 1, I) < 0 THEN BN(Z, K - 1, I) = 0
14050   NEXT K
14100   GOTO 14200
14150 WEND
14200 RETURN
14250 BN(Z, 2, I) = BN(Z, 1, I) + HN
14300 WHILE N > 1
14350   FOR K = 3 TO 2 * N - 1 STEP 2
14400     BN(Z, K, I) = BN(Z, K - 2, I) + WS
14450     BN(Z, K + 1, I) = BN(Z, K, I) + HN
14500     IF BN(Z, K, I) > 1 THEN BN(Z, K, I) = 1
14550     IF BN(Z, K + 1, I) > 1 THEN BN(Z, K + 1, I) = 1
14600   NEXT K
14650   GOTO 14750
14700 WEND
14750 RETURN
14800 REM *******************************************************************
14850 REM *  SUBROUTINE: CHECK SOLUTION                                         *
14900 REM *******************************************************************
14950 FOR Z = 1 TO NPRZON
15000   FOR I = 1 TO II - 1
15050   COUNT = 0
15100     FOR J = 1 TO JJ
15150       IF X(Z, J, I) = 1 THEN COUNT = COUNT + 1
15200     NEXT J
15250     WHILE COUNT <> 1
15300       LOCATE 19, 7: PRINT "INFEASIBLE SOLUTION! MAYBE IT CAN BE SOLVED"
15350       LOCATE 20, 7: PRINT "BY SHIFTING BOUNDS AT THE SAME WIDTH OR"
```

```
15400        LOCATE 21, 7: PRINT "ENLARGING REDUCTION FACTOR. TRY AGAIN!"
15450        GOTO 950
15500      WEND
15550   NEXT I
15600 NEXT Z
15650 RETURN
15660 REM **********************************************************************
15670 REM *  SUBROUTINE: PRINT OUTPUT FOR MODEL 5-9                             *
15680 REM **********************************************************************
15700 REM    **************************************************************
15755 REM    *  READ DATA FROM THE OUTPUT OF THE MIP MODEL, MIPOUT.DAT  *
15855 REM    **************************************************************
15900 II = 1
15910 TT = 1
16055 LINE INPUT £1, OUTVAL$
16105 WHILE MID$(OUTVAL$, 9, 2) = "N "
16155  I = VAL(MID$(OUTVAL$, 11, 2))       'grade
16205  H = VAL(MID$(OUTVAL$, 13, 2)) - 1 'total length of service has been
16255                                     'increased 1 year in XPRESS-MP model
16305  T = VAL(MID$(OUTVAL$, 15, 2))       'year
16355 N(I, H, T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)
16370 IF I > II THEN II = I
16380 IF T > TT THEN TT = T
16405 GOTO 18155
16455 WEND
16505 WHILE MID$(OUTVAL$, 9, 4) = "NR  "
16555  T = VAL(MID$(OUTVAL$, 15, 2))    'year
16605  NR(T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)
16655  GOTO 18155
16705 WEND
16755 WHILE MID$(OUTVAL$, 9, 2) = "NP"
16805  I = VAL(MID$(OUTVAL$, 11, 2))       'grade
16855  H = VAL(MID$(OUTVAL$, 13, 2)) - 1 'total length of service has been
16905                                     'increased 1 year in XPRESS-MP model
16955  T = VAL(MID$(OUTVAL$, 15, 2))       'year
17005  NP(I, H, T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)
17055  GOTO 18155
17105 WEND
17155 WHILE MID$(OUTVAL$, 9, 4) = "NREC"
17205  T = VAL(MID$(OUTVAL$, 15, 2))
17255  REC(T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)'recruitment cost
17305  GOTO 18155
17355 WEND
17405 WHILE MID$(OUTVAL$, 9, 4) = "NSKC"
17455  T = VAL(MID$(OUTVAL$, 15, 2))
17505  SKC(T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)'stock cost
17555  GOTO 18155
17605 WEND
17655 WHILE MID$(OUTVAL$, 9, 4) = "NLMC"
17705  T = VAL(MID$(OUTVAL$, 15, 2))
17755  LMC(T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)'lump sum cost
17805  GOTO 18155
17855 WEND
17905 WHILE MID$(OUTVAL$, 9, 4) = "NLFC"
17955  T = VAL(MID$(OUTVAL$, 15, 2))
18005  LFC(T) = INT(VAL(MID$(OUTVAL$, 28, 12)) + .5)'life pension cost
18055  GOTO 18155
```

```
18105 WEND
18155 IF NOT EOF(1) THEN 16055
18205 CLOSE £1
18255 REM *******************************************************************
18305 REM *  READ DATA FROM A FILE WHICH CONSISTS OF AVERAGE RECRUITMENT AGE, *
18355 REM *  AMO, RETIREMENT AGE, AR(i), MINIMUM REQUIRED TOTAL LENGTH OF     *
18405 REM *  SERVICE FOR PROMOTION, LRS(i), MINIMUM TOTAL LENGTH OF SERVICE   *
18455 REM *  IN EACH GRADE, LMI(i), AND INVOLUNTARY WASTAGE RATE. THESE DATA  *
18505 REM *  HAVEN'T BEEN INCREASED 1 YEAR.                                   *
18555 REM *******************************************************************
18605 OPEN "RIW.INP" FOR INPUT AS £2
18655 INPUT £2, AMO   'input average recruitment age
18760 LINE INPUT £2, INPVAL$        'input retirement age
18770   FOR I = 1 TO II
18780     AR(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
18790   NEXT I
18855 LINE INPUT £2, INPVAL$        'input LRS(i)
18905  FOR I = 1 TO II - 1
18955   LRS(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
19005  NEXT I
19055  LINE INPUT £2, INPVAL$       'input LMI(i)
19105 FOR I = 1 TO II
19155  LMI(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
19205 NEXT I
19455 FOR I = 1 TO II
19505  LMX(I) = AR(I) - AMO     'maximum total length of service
19555 NEXT I
19605 FOR I = 1 TO II
19655  C = 0
19705  LINE INPUT £2, INPVAL$   'input involuntary wastage rates
19755  BH = C * 10 + LMI(I) + 1 'the maximum number of data items in each row
19805                           'is 10. turnover starts from the next year of
19855                           'total length of service, LMI(i)
19905  WHILE MID$(INPVAL$, 71, 1) = "&"
19955   EH = (C + 1) * 10 + LMI(I)
20005   FOR H = BH TO EH
20055    RIW(I, H) = VAL(MID$(INPVAL$, (H - BH) * 7 + 1, 5))
20105   NEXT H
20155   C = C + 1
20205   GOTO 19705
20255  WEND
20305  EH = LMX(I)
20355  FOR H = BH TO EH
20405   RIW(I, H) = VAL(MID$(INPVAL$, (H - BH) * 7 + 1, 5))
20455  NEXT H
20505 NEXT I
20555 CLOSE £2
20605 REM *****************************************************************
20655 REM * READ INITIAL NUMBER OF STAFFS FROM FILES NI1.DAT TO NI6.DAT    *
20705 REM *****************************************************************
20755  FOR I = 1 TO II
20805   II1$ = STR$(I): II2$ = RIGHT$(II1$, LEN(II1$) - 1)
20855   FI$ = "NI" + II2$ + ".DAT"
20905   OPEN FI$ FOR INPUT AS £3
20955   C = 0
21005   LINE INPUT £3, INIDAT$
21055   BH = C * 10 + LMI(I)
```

```
21105   WHILE MID$(INIDAT$, 71, 1) = "&"
21155    EH = (C + 1) * 10 + LMI(I)
21205    FOR H = BH TO EH
21255     N(I, H, 0) = VAL(MID$(INIDAT$, (H - BH) * 7 + 1, 5))
21305    NEXT H
21355    C = C + 1
21405    GOTO 21005
21455  WEND
21505  EH = LMX(I) - 1
21555  FOR H = BH TO EH
21605    N(I, H, 0) = VAL(MID$(INIDAT$, (H - BH) * 7 + 1, 5))
21655  NEXT H
21705  CLOSE £3
21755 NEXT I
21805 REM ********************************************************************
21855 REM *  CALCULATE THE TOTAL NUMBER OF STAFFS IN GRADE, NA(I,T), TOTAL    *
21905 REM *  LENGTH OF SERVICE, NB(H,T), THE SYSTEM, NT(T), AND THE           *
21955 REM *  PERCENTAGE ON GRADE, PTGRD(I,T), AND TOTAL LENGTH OF SERVICE,    *
22005 REM *  PTTLS(H,T)                                                       *
22055 REM ********************************************************************
22105 FOR T = 0 TO TT
22155   FOR I = 1 TO II
22205     FOR H = LMI(I) TO LMX(I) - 1
22255       NA(I, T) = N(I, H, T) + NA(I, T)
22305     NEXT H
22355     NT(T) = NT(T) + NA(I, T)
22405   NEXT I
22455   FOR H = LMI(1) TO LMX(II) - 1
22505     FOR I = 1 TO II
22555       NB(H, T) = N(I, H, T) + NB(H, T)
22605     NEXT I
22655     PTTLS(H, T) = NB(H, T) / NT(T) * 100
22705   NEXT H
22755   FOR I = 1 TO II
22805     PTGRD(I, T) = NA(I, T) / NT(T) * 100
22855   NEXT I
22905 NEXT T
22955 REM ********************************************************************
23005 REM * CACULATE THE NUMBER OF PROMOTION IN GRADE I AT END OF YEAR T,    *
23055 REM * NPA(I,T), IN EACH GRADE AND TLS DURING TT YEARS, NPB(I,H), IN    *
23105 REM * EACH GRADE DURING TT YEARS, NPC(I), THE TOTAL NUMBER OF STAFF    *
23155 REM * IN GRADE I, NC(I), PROMOTION RATE IN GRADE I, TOTAL LENGTH OF    *
23205 REM * SERVICE H AT END OF YEAR T, RP(I,H,T), THE RATE IN GRADE I AT    *
23255 REM * END OF YEAR T, RPA(I,T), THE AVERAGE PROMOTION RATE IN EACH TLS *
23305 REM * AND GRADE DURING TT YEARS, RPB(I,H), AND THE AVERAGE PROMOTION  *
23355 REM * RATE IN EACH GRADE DURING TT YEARS, RPC(I)                      *
23405 REM ********************************************************************
23455 FOR T = 1 TO TT
23505   FOR I = 1 TO II - 1
23555     FOR H = LRS(I) TO LMX(I)
23605       RP(I, H, T) = NP(I, H, T) / (N(I, H - 1, T - 1) + .00001) * 100
23655       NPA(I, T) = NPA(I, T) + NP(I, H, T)
23705       NNA(I, T - 1) = NNA(I, T - 1) + N(I, H - 1, T - 1)
23755     NEXT H
23805     RPA(I, T) = NPA(I, T) / (NNA(I, T - 1) + .00001) * 100
23855   NEXT I
23905 NEXT T
```

```
23955 FOR I = 1 TO II                    'calculate the total number of staff in
24005   FOR T = 0 TO TT - 1              'each grade during 0 to TT-1 years for
24055     NNC(I) = NNC(I) + NNA(I, T) 'evaluating average total promotion rate
24105   NEXT T                           'in each grade
24155 NEXT I
24205 FOR I = 1 TO II - 1
24255   FOR H = LRS(I) TO LMX(I)
24305     FOR T = 1 TO TT
24355       NPB(I, H) = NPB(I, H) + NP(I, H, T)
24405       NNB(I, H - 1) = NNB(I, H - 1) + N(I, H - 1, T - 1)
24455     NEXT T
24505     RPB(I, H) = NPB(I, H) / (NNB(I, H - 1) + .00001) * 100
24555     NPC(I) = NPC(I) + NPB(I, H)
24605   NEXT H
24655   RPC(I) = NPC(I) / (NNC(I) + .00001) * 100
24670 NEXT I
24675 REM ***************************************************************
24677 REM *  WRITE RPB(I,H) INTO RPB.OUT FILE FOR CALCULATING THE         *
24680 REM *  PROBABILITY OF EVENTUAL PROMOTION AND EXPECTED WAITING TIME *
24685 REM ***************************************************************
24690 OPEN "RPB.OUT" FOR OUTPUT AS £5
24695 FOR I = 1 TO II - 1
24700   FOR H = LRS(I) TO LMX(I)
24705     IF H = LMX(I) THEN PRINT £5, USING "£.££££"; RPB(I, H) / 100: GOTO 24715
24710     PRINT £5, USING "£.££££"; RPB(I, H) / 100; : PRINT £5, ", ";
24715   NEXT H
24720 NEXT I
24730 CLOSE £5
24732 LOCATE 5, 4:PRINT "1. AVERAGE PROMOTION RATES FOR GRADE i WITH TOTAL LENGTH"
24735 LOCATE 6, 7:PRINT "OF SERVICE h, RPB(i,h), HAVE BEEN WRITTEN INTO"
24740 LOCATE 7, 7:PRINT "RPB.OUT FILE"
24755 REM ******************************************************************
24805 REM * CALCULATE THE NUMBER OF WASTAGE, NWGA(I,H,T), NWGB(I,T),        *
24855 REM * NWGC(H,T), NWGT(T)                                              *
24905 REM ******************************************************************
24955 FOR T = 1 TO TT
25005   FOR I = 1 TO II
25055     FOR H = LMI(I) + 1 TO LMX(I)
25105       NWGA(I, H, T) = INT(RIW(I, H) * N(I, H - 1, T - 1) + .5)
25155       NWGB(I, T) = NWGB(I, T) + NWGA(I, H, T)
25205     NEXT H
25255     NWGT(T) = NWGT(T) + NWGB(I, T)
25305   NEXT I
25355 NEXT T
25405 FOR T = 1 TO TT
25455   FOR H = LMI(1) + 1 TO LMX(II)
25505     FOR I = 1 TO II
25555       NWGC(H, T) = NWGC(H, T) + NWGA(I, H, T)
25605     NEXT I
25655   NEXT H
25705 NEXT T
25755 REM ******************************************************************
25805 REM * CALCULATE THE NUMBER OF RETIREMENT, NRT(I,T), NRTA(T), NRTB(I), *
25855 REM * NRTT                                                            *
25905 REM ******************************************************************
25955 FOR T = 1 TO TT
26005   FOR I = 1 TO II
```

```
26055     NRT(I, T) = N(I, LMX(I), T)
26105     NRTA(T) = NRTA(T) + NRT(I, T)
26155   NEXT I
26205   NRTT = NRTT + NRTA(T)
26255 NEXT T
26305 FOR I = 1 TO II
26355   FOR T = 1 TO TT
26405     NRTB(I) = NRTB(I) + NRT(I, T)
26455   NEXT T
26505 NEXT I
26555 REM ****************************************************************
26605 REM * WRITE THE NUMBER OF STAFF INTO A DISK FILE, OUTPUT.TAB          *
26655 REM ****************************************************************
26705 OPEN "OUTPUT.TAB" FOR OUTPUT AS £4
26755 FOR T = 0 TO TT
26805 PRINT £4, TAB(24); "Number of Staff - End of Year "; T
26855 FOR I = 1 TO 78: PRINT £4, "_"; : NEXT I: PRINT £4,
26905 PRINT £4, TAB(30); "Grade"
26955 PRINT £4, "Total Length"; TAB(14); : FOR I = 1 TO 47: PRINT £4, "_"; : NEXT I
27005 PRINT £4, : PRINT £4, TAB(68); "Percentage"
27055 PRINT £4, "of Service"; : FOR I=1 TO 6: PRINT £4, TAB((I - 1)* 8+15); I; : NEXT I
27105 PRINT £4, TAB(62); "Total"; TAB(74); "(%)"
27155 FOR I = 1 TO 78: PRINT £4, "_"; : NEXT I: PRINT £4,
27205   FOR H = LMI(1) TO LMX(II) - 1
27255     PRINT £4, USING "££"; TAB(5); H;
27305     FOR I = 1 TO II
27320     IF H < LMI(I) OR H >= LMX(I) THEN GOTO 27405
27355       PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 14); N(I, H, T);
27405     NEXT I
27455     PRINT £4, USING "££££££"; TAB(62); NB(H, T);
27505     PRINT £4, USING "££.££"; TAB(71); PTTLS(H, T)
27555   NEXT H
27605 FOR I = 1 TO 78: PRINT £4, "_"; : NEXT I: PRINT £4,
27655 PRINT £4, "Total";
27705   FOR I = 1 TO II
27755     PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 14); NA(I, T);
27805   NEXT I
27855 PRINT £4, USING "££££££"; TAB(62); NT(T)
27905 PRINT £4, "Percentage(%)";
27955   FOR I = 1 TO II
28005     PRINT £4, USING "££.££"; TAB((I - 1) * 8 + 16); PTGRD(I, T);
28055   NEXT I
28105   PRINT £4,
28155 FOR I = 1 TO 78: PRINT £4, "_"; : NEXT I: PRINT £4, : PRINT £4,
28205 NEXT T
28255 REM ****************************************************************
28305 REM * WRITE THE NUMBER OF PROMOTION INTO  A DISK  FILE, OUTPUT.TAB    *
28355 REM ****************************************************************
28405 FOR T = 1 TO TT
28455 PRINT £4, TAB(9); "Number of Promotion - End of Year "; T
28505 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
28555 PRINT £4, TAB(30); "Grade"
28605 PRINT £4, "Total Length"; TAB(14); : FOR I = 1 TO 40: PRINT £4, "_"; : NEXT I
28655 PRINT £4,
28705 PRINT £4, "of Service"; : FOR I = 1 TO II - 1
28755 PRINT £4, TAB((I - 1) * 8 + 15); I; : NEXT I
28805 PRINT £4,
```

```
28855 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
28905   FOR H = LRS(1) TO LMX(II - 1)
28955     PRINT £4, USING "££"; TAB(5); H;
29005     FOR I = 1 TO II - 1
29020     IF H < LRS(I) OR H > LMX(I) THEN GOTO 29105
29055       PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 14); NP(I, H, T);
29105     NEXT I
29155     PRINT £4,
29205   NEXT H
29255 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
29305 PRINT £4, "Total";
29355   FOR I = 1 TO II - 1
29405     PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 14); NPA(I, T);
29455   NEXT I
29505   PRINT £4,
29555   FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4, : PRINT £4,
29605 NEXT T
29655 REM *****************************************************************
29705 REM * WRITE THE PROMOTION RATES INTO  A DISK  FILE, OUTPUT.TAB          *
29755 REM *****************************************************************
29805 FOR T = 1 TO TT
29855 PRINT £4, TAB(12); "Promotion Rates (%) - End of Year "; T
29905 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
29955 PRINT £4, TAB(30); "Grade"
30005 PRINT £4, "Total Length"; TAB(14); : FOR I = 1 TO 40: PRINT £4, "_"; : NEXT I
30055 PRINT £4,
30105 PRINT £4, "of Service"; : FOR I = 1 TO II - 1
30155 PRINT £4, TAB((I - 1) * 8 + 15); I; : NEXT I
30205 PRINT £4,
30255 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
30305   FOR H = LRS(1) TO LMX(II - 1)
30355     PRINT £4, USING "££"; TAB(5); H;
30405     FOR I = 1 TO II - 1
30420     IF H < LRS(I) OR H > LMX(I) THEN 30505
30455       PRINT £4, USING "£££.££"; TAB((I - 1) * 8 + 14); RP(I, H, T);
30505     NEXT I
30555     PRINT £4,
30605   NEXT H
30655 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
30705 PRINT £4, "Total";
30755   FOR I = 1 TO II - 1
30805     PRINT £4, USING "£££.££"; TAB((I - 1) * 8 + 14); RPA(I, T);
30855   NEXT I
30905   PRINT £4,
30955   FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4, : PRINT £4,
31005 NEXT T
31055 REM *****************************************************************
31105 REM * WRITE AVERAGE PROMOTION RATES IN TT YEARS INTO  THE DISK  FILE,  *
31155 REM * OUTPUT.TAB                                                      *
31205 REM *****************************************************************
31255 PRINT £4, TAB(6); "Average Promotion Rates of T Years (%)"
31305 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
31355 PRINT £4, TAB(30); "Grade"
31405 PRINT £4, "Total Length"; TAB(14); : FOR I = 1 TO 40: PRINT £4, "_"; : NEXT I
31455 PRINT £4,
31505 PRINT £4, "of Service"; : FOR I = 1 TO II - 1
31555 PRINT £4, TAB((I - 1) * 8 + 15); I; : NEXT I
```

```
31605 PRINT £4,
31655 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
31705   FOR H = LRS(1) TO LMX(II - 1)
31755     PRINT £4, USING "££"; TAB(5); H;
31805     FOR I = 1 TO II - 1
31820     IF H < LRS(I) OR H > LMX(I) THEN 31905
31855       PRINT £4, USING "£££.££"; TAB((I - 1) * 8 + 14); RPB(I, H);
31905     NEXT I
31955     PRINT £4,
32005   NEXT H
32055 FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4,
32105 PRINT £4, "Average";
32155   FOR I = 1 TO II - 1
32205     PRINT £4, USING "££.££"; TAB((I - 1) * 8 + 14); RPC(I);
32255   NEXT I
32305   PRINT £4,
32355   FOR I = 1 TO 54: PRINT £4, "_"; : NEXT I: PRINT £4, : PRINT £4,
32405 REM ****************************************************************
32455 REM * WRITE THE NUMBER OF WASTAGE INTO  A DISK  FILE, OUTPUT.TAB      *
32505 REM ****************************************************************
32555 FOR T = 1 TO TT
32605   PRINT £4, TAB(21); "Number of Wastage - End of Year "; T
32655   FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
32705   PRINT £4, TAB(30); "Grade"
32755   PRINT £4, "Total Length"; TAB(14); : FOR I = 1 TO 47: PRINT £4, "_"; : NEXT I
32805   PRINT £4,
32855   PRINT £4, "of Service";
32905   FOR I = 1 TO 6
32955     PRINT £4, TAB((I - 1) * 8 + 15); I;
33005   NEXT I
33055   PRINT £4, TAB(62); "Total"
33105   FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
33155   FOR H = LMI(1) + 1 TO LMX(II)
33205     PRINT £4, USING "££"; TAB(5); H;
33255     FOR I = 1 TO II
33270     IF H < (LMI(I) + 1) OR H > LMX(I) THEN 33355
33305       PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 12); NWGA(I, H, T);
33355     NEXT I
33405     PRINT £4, USING "££££££"; TAB(62); NWGC(H, T)
33455   NEXT H
33505   FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
33555   PRINT £4, "Total";
33605   FOR I = 1 TO II
33655     PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 12); NWGB(I, T);
33705   NEXT I
33755   PRINT £4, USING "££££££"; TAB(62); NWGT(T)
33805   FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4, : PRINT £4,
33855 NEXT T
33905 REM ****************************************************************
33955 REM * WRITE THE NUMBER OF RETIREMENT INTO  A DISK  FILE, OUTPUT.TAB   *
34005 REM ****************************************************************
34055 PRINT £4, TAB(23); "Number of Retirement"
34105 FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
34155 PRINT £4, TAB(30); "Grade"
34205 PRINT £4, TAB(14); : FOR I = 1 TO 47: PRINT £4, "_"; : NEXT I
34255 PRINT £4,
34305 PRINT £4, "Year"; : FOR I = 1 TO 6: PRINT £4, TAB((I - 1) * 8 + 15); I; : NEXT I
```

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34355 PRINT £4, TAB(62); "Total"
34405 FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
34455   FOR T = 1 TO TT
34505     PRINT £4, USING "££"; TAB(2); T;
34555     FOR I = 1 TO II
34605       PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 12); NRT(I, T);
34655     NEXT I
34705     PRINT £4, USING "££££££"; TAB(60); NRTA(T)
34755   NEXT T
34805 FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I: PRINT £4,
34855 PRINT £4, "Total";
34905   FOR I = 1 TO II
34955     PRINT £4, USING "££££££"; TAB((I - 1) * 8 + 12); NRTB(I);
35005   NEXT I
35055   PRINT £4, USING "££££££"; TAB(60); NRTT
35105   FOR I = 1 TO 68: PRINT £4, "_"; : NEXT I:
35160 LOCATE 8, 4: PRINT "2. THE NUMBER OF STAFF, THE NUMBER OF PROMOTION, PROMOTION"
35165 LOCATE 9, 7: PRINT "RATES, AVERAGE PROMOTION RATES, THE NUMBER OF WASTAGE"
35170 LOCATE 10, 7: PRINT "AND THE NUMBER OF RETIREMENT HAVE BEEN WRITTEN INTO"
35180 LOCATE 11, 7: PRINT "OUTPUT.TAB FILE"
35200 CLOSE £4
35205 REM ******************************************************************
35255 REM * REORGANIZE THE OUTPUT AND WRITE IT INTO DISK FILES FOR PLOTING  *
35305 REM * GRAPHS                                                          *
35355 REM ******************************************************************
35405 OPEN "MPN.GRF" FOR OUTPUT AS £7
35655 PRINT £7, TAB(15); "Table 1. Number of Staff": PRINT £7, : PRINT £7,
35705 PRINT £7, TAB(2); "Year"; TAB(11); "I"; TAB(22); "II"; TAB(33); "III";
35755 PRINT £7, TAB(46); "IV"; TAB(59); "V"; TAB(70); "VI"
35805 FOR T = 1 TO TT
35855 PRINT £7, USING "££"; TAB(2); T;
35905   FOR I = 1 TO II
35955   PRINT £7, USING "££££££"; TAB(6 + (I - 1) * 12); NA(I, T);
36005   NEXT I
36055 PRINT £7,
36105 NEXT T
36110 FOR L = 1 TO 240 - TT: PRINT £7, " ": NEXT L
36115 CLOSE £7
36120 LOCATE 12, 4: PRINT "3. NUMBER OF STAFF FOR GRAPH IS IN MPN.GRF FILE"
36125 OPEN "MPNP.GRF" FOR OUTPUT AS £8
36155 PRINT £8, TAB(15); "Table 2. Number of Promotion": PRINT £8, : PRINT £8,
36205 PRINT £8, TAB(2); "Year"; TAB(11); "I"; TAB(22); "II"; TAB(33); "III";
36255 PRINT £8, TAB(46); "IV"; TAB(59); "V"
36305 FOR T = 1 TO TT
36355 PRINT £8, USING "££"; TAB(2); T;
36405   FOR I = 1 TO II - 1
36455   PRINT £8, USING "££££££"; TAB(6 + (I - 1) * 12); NPA(I, T);
36505   NEXT I
36555 PRINT £8,
36605 NEXT T
36710 FOR L = 1 TO 240 - TT: PRINT £8, " ": NEXT L
36715 CLOSE £8
36720 LOCATE 13, 4: PRINT "4. NUMBER OF PROMOTION FOR GRAPH IS IN MPNP.GRF FILE"
36725 OPEN "MPNT.GRF" FOR OUTPUT AS £9
36755 PRINT £9, TAB(15); "Table 3. Total Staff": PRINT £9, : PRINT £9,
36805 PRINT £9, TAB(13); "Year"; TAB(20); "Staff"
36855 FOR T = 1 TO TT
```

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36905 PRINT £9, USING "££"; TAB(14); T;
36955 PRINT £9, USING "£££££"; TAB(19); NT(T)
37005 NEXT T
37010 FOR L = 1 TO 240 - TT: PRINT £9, " ": NEXT L
37015 CLOSE £9
37020 LOCATE 14, 4: PRINT "5. TOTAL STAFF FOR GRAPH IS IN MPNT.GRF FILE"
37025 OPEN "MPNR.GRF" FOR OUTPUT AS £10
37055 PRINT £10, TAB(15); "Table 4. Number of Recruits": PRINT £10, : PRINT £10,
37105 PRINT £10, TAB(13); "Year"; TAB(20); "Recruit"
37155 FOR T = 1 TO TT
37205 PRINT £10, USING "££"; TAB(14); T;
37255 PRINT £10, USING "£££££"; TAB(21); NR(T)
37305 NEXT T
37310 FOR L = 1 TO 240 - TT: PRINT £10, " ": NEXT L
37315 CLOSE £10
37320 LOCATE 15, 4: PRINT "6. NUMBER OF RECRUITMENT FOR GRAPH IS IN MPNR.GRF FILE"
37325 OPEN "MPCOST.GRF" FOR OUTPUT AS £11
37355 FOR T = 1 TO TT
37405 TCOST(T) = REC(T) + SKC(T) + LMC(T) + LFC(T)'total cost
37455 NEXT T
37505 PRINT £11, TAB(15); "Table 5. Manpower Cost: £K": PRINT £11, : PRINT £11,
37555 PRINT £11, TAB(2); "Year"; TAB(12); "Stock"; TAB(22); "Recruitment";
37605 PRINT £11, TAB(38); "Lump Sum"; TAB(51); "Pension"
37655 FOR T = 1 TO TT
37705 PRINT £11, USING "££"; TAB(2); T;
37755 PRINT £11, USING "££££££££"; TAB(9); SKC(T); TAB(23); REC(T);
37805 PRINT £11, USING "££££££££"; TAB(37); LMC(T); TAB(50); LFC(T)
37855 NEXT T
37860 FOR L = 1 TO 240 - TT: PRINT £11, " ": NEXT L
37865 CLOSE £11
37870 LOCATE 16, 4: PRINT "7. MANPOWER COST FOR GRAPHS IS IN MPCOST.GRF FILE"
38055 REM ***************************************************************
38105 REM * REORGANISE PROMOTION RATES FOR PLOTING GRAPH AND WRITE THEM      *
38155 REM * INTO DISK FILES                                                 *
38205 REM ***************************************************************
38210 OPEN "MPRP.GRF" FOR OUTPUT AS £17
38215 PRINT £17, TAB(15); "Table 6. Promotion Rates (%)": PRINT £17, : PRINT £17,
38225 PRINT £17, TAB(2); "Year"; TAB(9); "I"; TAB(16); "II"; TAB(23); "III";
38230 PRINT £17, TAB(30); "IV"; TAB(37); "V"
38240 FOR T = 1 TO TT
38250 PRINT £17, USING "££"; TAB(3); T;
38260   FOR I = 1 TO II - 1
38270   PRINT £17, USING "££.£"; TAB(7 + (I - 1) * 7); RPA(I, T);
38280   NEXT I
38290 PRINT £17,
38300 NEXT T
38310 FOR L = 1 TO 240 - TT: PRINT £17, " ": NEXT L
38320 CLOSE £17
38330 LOCATE 17, 4: PRINT "8. PROMOTION RATES FOR GRAPH IS IN MPRP.GRF FILE"
38340 LOCATE 20, 30: PRINT "FINISH!"
38400 RETURN
39105 REM ********************************************************
39120 REM *  SUBROUTINE:                                         *
39155 REM *  READ SOLUTION OF MODEL 3-18 FROM MIPOUT.DAT FILE     *
39205 REM ********************************************************
40000 II = 1        'number of grade
40100 TT = 1        'planning periods
```

```
40750 LINE INPUT £1, OUTVAL$
40760 I = VAL(MID$(OUTVAL$, 13, 2))          'grade
40765 T = VAL(MID$(OUTVAL$, 15, 2))          'year
40766 IF I > II THEN II = I
40768 IF T > TT THEN TT = T
40770 IF MID$(OUTVAL$,9,4)= "NN  " THEN MPN(I,T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40775 IF MID$(OUTVAL$,9,4)= "NP  " THEN MPNP(I,T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40780 IF MID$(OUTVAL$,9,4)= "NT  " THEN MPNT(T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40785 IF MID$(OUTVAL$,9,4)= "NR  " THEN MPNR(I,T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40790 IF MID$(OUTVAL$,9,4)= "NSKC" THEN MPSC(T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40795 IF MID$(OUTVAL$,9,4)= "NREC" THEN MPRC(T) = VAL(MID$(OUTVAL$, 28, 12)): GOTO 40805
40800 IF MID$(OUTVAL$,9,4)= "NPEC" THEN MPPC(T) = VAL(MID$(OUTVAL$, 28, 12))
40805 IF NOT EOF(1) THEN 40750
40810 RETURN
40811 REM ***************************************************************
40812 REM *  SUBROUTINE:                                                 *
40813 REM *  REORGANISE OUTPUT OF MODEL 3-18 FOR PLOTING GRAPHICS        *
40814 REM ***************************************************************
40815 FOR Z = 1 TO NPRZON
40820   FOR I = 1 TO II - 1
40825     FOR J = 1 TO JJ
40830      WHILE X(Z, J, I) = 1
40835      BL1(I) = B(Z, 2 * J - 1, I)
40840      BU1(I) = B(Z, 2 * J, I)
40845      GOTO 40855
40850      WEND
40855     NEXT J
40860   NEXT I
40865 NEXT Z
40870 OPEN "MPNI" FOR INPUT AS £2
40880 FOR I = 1 TO II
40885 WHILE EOF(2)
40890 GOSUB 45510: SYSTEM
40895 WEND
40900 INPUT £2, MPN(I, 0)
40905 NEXT I
40910 WHILE NOT EOF(2)
40915 GOSUB 45510: SYSTEM
40920 WEND
40925 CLOSE £1, £2
40930 REM *****************************************************
40935 REM *  REORGANIZE OUTPUT DATA AND WRITE INTO DISK        *
40940 REM *  FILES FOR PLOTING GRAPHICS                       *
40945 REM *****************************************************
40950 OPEN "MPN.GRF" FOR OUTPUT AS £7
40955 OPEN "MPNP.GRF" FOR OUTPUT AS £8
40960 OPEN "MPNT.GRF" FOR OUTPUT AS £9
40965 OPEN "MPNR.GRF" FOR OUTPUT AS £10
40970 OPEN "MPCOST.GRF" FOR OUTPUT AS £11
40975 PRINT £7, TAB(15); "Table 1. Number of staff": PRINT £7, : PRINT £7,
40980 PRINT £7, TAB(2); "Year"; TAB(10); "I"; TAB(20); "II"; TAB(30); "III";
40985 PRINT £7, TAB(40); "IV"; TAB(51); "V"; TAB(60); "VI"
40990 FOR T = 1 TO TT
40995 PRINT £7, USING "££"; TAB(2); T;
41000   FOR I = 1 TO II
41005   PRINT £7, USING "£££££"; TAB(8 + (I - 1) * 10); MPN(I, T);
41010   NEXT I
```

```
41020 PRINT £7,
41030 NEXT T
41040 PRINT £8, TAB(15); "Table 2. Number of promotion": PRINT £8, : PRINT £8,
41050 PRINT £8, TAB(2); "Year"; TAB(9); "NP12"; TAB(21); "NP23"; TAB(33); "NP34";
41100 PRINT £8, TAB(45); "NP45"; TAB(57); "NP56"
41200 FOR T = 1 TO TT
41300 PRINT £8, USING "££"; TAB(2); T;
41400   FOR I = 1 TO II - 1
41500   PRINT £8, USING "£££££"; TAB(8 + (I - 1) * 12); MPNP(I, T);
41600   NEXT I
41700 PRINT £8,
41800 NEXT T
41900 PRINT £8, : PRINT £8,
42000 PRINT £8, TAB(3); "NPij= number of staff promoted from grade i to j"
42100 PRINT £9, TAB(15); "Table 3. Total staff": PRINT £9, : PRINT £9,
42150 PRINT £9, TAB(13); "Year"; TAB(20); "Total staff"
42200 FOR T = 1 TO TT
42300 PRINT £9, USING "££"; TAB(13); T;
42500 PRINT £9, USING "£££££"; TAB(22); MPNT(T)
42600 NEXT T
42700 PRINT £10, TAB(15); "Table 4. Number of recruit": PRINT £10, : PRINT £10,
42800 PRINT £10, TAB(13); "Year"; TAB(20); "Number of recruit"
43000 FOR T = 1 TO TT
43100 PRINT £10, USING "££"; TAB(13); T;
43200 PRINT £10, USING "£££££"; TAB(23); MPNR(1, T)
43300 NEXT T
43310 FOR T = 1 TO TT
43320 TC(T) = MPSC(T) + MPRC(T) + MPPC(T)  'total cost
43330 NEXT T
43400 PRINT £11, TAB(15); "Table 5. Manpower cost: £K": PRINT £11, : PRINT £11,
43500 PRINT £11, TAB(2); "Year"; TAB(8); "Stock cost"; TAB(22); "Recruitment cost";
43600 PRINT £11, TAB(40); "Pension cost"; TAB(56); "Total cost"
43800 FOR T = 1 TO TT
43900 PRINT £11, USING "££"; TAB(2); T;
43910 PRINT £11, USING "£££££££"; TAB(8); MPSC(T); TAB(22); MPRC(T);
44000 PRINT £11, USING "£££££££"; TAB(40); MPPC(T); TAB(56); TC(T)
44100 NEXT T
44130 FOR L = 1 TO 240 - TT: PRINT £7, " ": PRINT £8, " ": PRINT £9, " "
44140 PRINT £10, " ": PRINT £11, " ": NEXT L
44150 CLOSE £7, £8, £9, £10, £11
44210 REM *****************************************************
44220 REM *  CALCULATE PROJECTED PROMOTION RATES AND           *
44230 REM *  REORGANIZE THEM FOR PLOTING GRAPHICS              *
44240 REM *****************************************************
44300 OPEN "MPRP1.GRF" FOR OUTPUT AS £11
44310 OPEN "MPRP2.GRF" FOR OUTPUT AS £12
44320 OPEN "MPRP3.GRF" FOR OUTPUT AS £13
44330 OPEN "MPRP4.GRF" FOR OUTPUT AS £14
44340 OPEN "MPRP5.GRF" FOR OUTPUT AS £15
44500 FOR I = 1 TO II - 1
44510 PRINT £I+10,TAB(15); "Table"; I + 4; " Promotion rate ( grade"; I; " to"; I + 1; ")"
44520 PRINT £I + 10, :  PRINT £I + 10,
44530 PRINT £I+10, TAB(8); "Year"; TAB(15); "Lower bound"; TAB(35); "Project";
44540 PRINT £I+10, TAB(55); "Upper bound"
44700   FOR T = 1 TO TT
44900   RP1(I, T) = MPNP(I, T) / (MPN(I, T - 1) + .00001)
45000   PRINT £I + 10, USING "££"; TAB(8); T;
```

```
45010   PRINT £I + 10, USING "£.£££"; TAB(16); BL1(I); TAB(35); RP1(I, T); TAB(56); BU1(I)
45100   NEXT T
45300 NEXT I
45330 FOR L = 1 TO 240 - TT: PRINT £11, " ": PRINT £12, " ": PRINT £13, " "
45340 PRINT £14, " ": PRINT £15, " ": NEXT L
45400 CLOSE £11, £12, £13, £14, £15
45500 RETURN
45502 REM ************************************************************
45504 REM *   SUBROUTINE : PRINT THE ERROR MESSAGE OF READING BINARY     *
45506 REM *                VARIABLES FROM INPUT FILE                     *
45508 REM ************************************************************
45510 CLS
45515 PRINT : PRINT : PRINT : PRINT
45520 LOCATE 5, 16: PRINT "ERROR ! THE FILE OF INITIAL NUMBER OF STAFF DOES  "
45525 LOCATE 6, 16: PRINT "NOT MATCH THE DATA OF INPUT, THAT IS,"
45530 LOCATE 7, 16: PRINT "THE NUMBER OF GRADE."
45535 RETURN
```

APPENDIX C4 - PROGRAM FOR CALCULATING THE PROBABILITY AND EXPECTED WAITING TIME BEFORE PROMOTION

```
100    II = 6 'the number of grade
200    DIM RPB(6, 30), RIW(6, 30)
220    DIM AR(6), LRS(6), LMI(6), LMX(6)
1000   REM *******************************************************************
1050   REM *  READ DATA FROM A FILE WHICH CONSISTS OF AVERAGE RECRUITMENT AGE, *
1100   REM *  AMO, RETIREMENT AGE, AR(i), MINIMUM REQUIRED TOTAL LENGTH OF     *
1110   REM *  SERVICE FOR PROMOTION, LRS(i), MINIMUM TOTAL LENGTH OF SERVICE   *
1150   REM *  IN EACH GRADE, LMI(i), AND INVOLUNTARY WASTAGE RATE. THESE DATA  *
1170   REM *  HAVEN'T BEEN INCREASED 1 YEAR.                                   *
1200   REM *******************************************************************
1300   OPEN "RIW.INP" FOR INPUT AS £1
1350   INPUT £1, AMO   'input average recruitment age
1400   LINE INPUT £1, INPVAL$         'input retirement age
1450     FOR I = 1 TO II
1500       AR(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
1550     NEXT I
1600   LINE INPUT £1, INPVAL$         'input LRS(i)
1650    FOR I = 1 TO II - 1
1700     LRS(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
1750    NEXT I
1800    LINE INPUT £1, INPVAL$        'input LMI(i)
1850   FOR I = 1 TO II
1900    LMI(I) = VAL(MID$(INPVAL$, (I - 1) * 4 + 1, 2))
1950   NEXT I
2000   FOR I = 1 TO II
2050    LMX(I) = AR(I) - AMO      'maximum total length of service
2100   NEXT I
2150   FOR I = 1 TO II
2200    C = 0
2250    LINE INPUT £1, INPVAL$   'input involuntary wastage rates
2300    BH = C * 10 + LMI(I) + 1 'the maximum number of data items in each row
2350                             'is 10. turnover starts from the next year of
2400                             'total length of service, LMI(i)
2450    WHILE MID$(INPVAL$, 71, 1) = "&"
2500     EH = (C + 1) * 10 + LMI(I)
2550     FOR H = BH TO EH
2600      RIW(I, H) = VAL(MID$(INPVAL$, (H - BH) * 7 + 1, 5))
2650     NEXT H
2700     C = C + 1
2750     GOTO 2250
2800    WEND
2850    EH = LMX(I)
2900    FOR H = BH TO EH
2950     RIW(I, H) = VAL(MID$(INPVAL$, (H - BH) * 7 + 1, 5))
3000    NEXT H
3050   NEXT I
3100   CLOSE £1
3150 REM **************************************************************
3160 REM *  READ AVERAGE PROMOTION RATES OF T YEARS, RPB(I,H), FROM    *
3170 REM *  RPB.OUT FILE                                              *
3180 REM **************************************************************
```

```
3200  OPEN "RPB.OUT" FOR INPUT AS £2
3250  FOR I = 1 TO II - 1
3300    FOR H = LRS(I) TO LMX(I)
3350      INPUT £2, RPB(I, H)
3400    NEXT H
3450  NEXT I
3500  CLOSE £2
3776 REM ** P(N,G,J) -  n-step transition probability for grade 1 to grade 6
3778 DIM P(30, 30, 6)
3784 DIM PU(5, 30, 6)  'the probability of eventual promotion to a specified grade
3786 DIM EW(5, 30, 6)  'the expected waiting time before eventual promotion
3790 DIM REMAIN(6, 30) 'the probability of remaining in the same grade
3792 REM ************************************************************
3793 REM *  CALCULATE THE PROBABILITY OF EVENTUAL PROMOTION AND EXPECTED *
3795 REM *  WAITING TIME, AND WRITE THEM INTO PROB.TAB FILE             *
3797 REM ************************************************************
3800 FOR I = 1 TO II
3802   FOR H = LMI(I) TO LMX(I) - 1
3803     IF I = II THEN REMAIN(I, H + 1) = 1 - RIW(I, H + 1): GOTO 3806'equation (6-1c)
3804     REMAIN(I, H + 1) = 1 - RPB(I, H + 1) - RIW(I, H + 1)'equation (6-1b)
3806   NEXT H
3808 NEXT I
3809 OPEN "EPRO&WT.TAB" FOR OUTPUT AS £3
3810 I = 1: GOSUB 5500: GOSUB 45920
3815 I = 2: ERASE P: GOSUB 5500: GOSUB 45920
3820 I = 3: ERASE P: GOSUB 5500: GOSUB 45920
3830 I = 4: ERASE P: GOSUB 5500: GOSUB 45920
3840 I = 5: ERASE P: GOSUB 5500: GOSUB 45920
3975 CLOSE £3
3980 SYSTEM
5000 REM ************************************************************
5100 REM *  CALCULATE THE PROBABILITY OF EVENTUAL PROMOTION TO A        *
5200 REM *  SPECIFIED GRADE AND THE EXPECTED WAITING TIME FOR PROMOTION  *
5400 REM ************************************************************
5500 FOR G = LMI(I) TO LMX(I) - 1
5600   P(1, G, I) = REMAIN(I, G + 1)  'equation (6-2b)
5700   P(1, G, I + 1) = RPB(I, G + 1) 'equation (6-2c)
5800 NEXT G
5900 FOR G=LMI(I) TO LMX(I)-1
6000   FOR N=2 TO LMX(I)-G
6100     P(N,G,I)=P(N-1,G,I)*REMAIN(I,G+N)  '(6-2e)
6200   NEXT N
6300 NEXT G
6400 FOR G=LMI(I) TO LMX(I)-1
6500   FOR J = I + 1 TO II
6600     IF J - I > LMI(J) - G THEN K = J - I: GOTO 6800
6700     K = LMI(J) - G
6800     IF K < 2 THEN K = 2
6900     FOR N = K TO LMX(J) - G
7000       IF N >= (LMX(J - 1) - G + 1) THEN RPB(J - 1, G + N) = 0
7100       IF N <= (LMI(J) - G) THEN REMAIN(J, G + N) = 0
7200       P(N,G,J)=P(N-1,G,J-1)*RPB(J-1,G+N) + P(N - 1, G, J) * REMAIN(J, G + N) '(6-2d)
7300     NEXT N
7400   NEXT J
7500 NEXT G
```

```
7550 REM ******************************************************************
7600 REM *  the probability of an individual ultimately being promoted to   *
7650 REM *  grade j, given that at current time (year 0) the individual is  *
7700 REM *  in grade i with total length of service g, i.e. equation (6-6)  *
7800 REM ******************************************************************
8000 FOR G=LMI(I) TO LMX(I)-1
8100   FOR J = I + 1 TO II
8200     IF J - I > LMI(J) - G THEN K = J - I: GOTO 8400
8300     K = LMI(J) - G
8400     FOR N = K TO LMX(J-1) - G
8500       IF N=1 THEN P(N-1,G,J - 1) = 1
8600       PU(I,G, J) = PU(I,G,J)+P(N - 1, G, J - 1) * RPB(J - 1, G + N) '(6-6)
8700     NEXT N
8800   NEXT J
8900 NEXT G
8920 REM ******************************************************************
8930 REM *  expected waiting time of eventual promotion, EW(I,G,J)          *
8950 REM ******************************************************************
9000 FOR G=LMI(I) TO LMX(I)-1
9100   FOR J = I + 1 TO II
9200     IF J - I > LMI(J) - G THEN K = J - I: GOTO 9400
9300     K = LMI(J) - G
9400     FOR N = K TO LMX(J-1) - G
9500       IF N=1 THEN P(N-1,G,J - 1) = 1
9600       EW(I,G,J)= EW(I,G,J)+N*P(N - 1, G, J - 1) * RPB(J - 1, G + N)/PU(I,G,J) '(6-6)
9700     NEXT N
9800   NEXT J
9900 NEXT G
10000 RETURN
45895 REM ******************************************************************
45896 REM *  WRITE THE PROBABILITY OF EVENTUAL PROMOTION TO A SPECIFIED GRADE *
45897 REM *  INTO THE DISK FILE, EPRO&WT.TAB                                  *
45900 REM ******************************************************************
45920 PRINT £3, TAB(8); "The Probability of Eventual Promotion (%)"
45922 PRINT £3, TAB(22); "from Grade"; I
45925 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3,
45930 PRINT £3, TAB(19); "Eventual Promotion to Grade"
45940 PRINT £3, "Total Length"; TAB(15); : FOR F = 1 TO 41: PRINT £3, "_"; : NEXT F
45945 PRINT £3,
45950 PRINT £3, "of Service"; : FOR F = 2 TO II
45960 PRINT £3, TAB((F - 2) * 8 + 16); F; : NEXT F
45965 PRINT £3,
45970 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3,
45980   FOR G = LMI(I) TO LMX(I) - 1
45990     PRINT £3, USING "££"; TAB(5); G;
46000     FOR J = I + 1 TO II
46010       PU(I, G, J) = PU(I, G, J) * 100
46030       PRINT £3, USING "£££.££"; TAB((J - 1) * 8 + 6); PU(I, G, J);
46040     NEXT J
46050     PRINT £3,
46060   NEXT G
46070 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3, : PRINT £3,
46085 PRINT £3,
46090 REM ******************************************************************
46100 REM *  WRITE THE EXPECTED WAITING TIME FOR PROMOTION INTO THE DISK      *
46110 REM *  FILE, EPRO&WT.TAB                                                *
46120 REM ******************************************************************
```

```
46140 PRINT £3, TAB(9); "The Expected Waiting Time for Promotion"
46145 PRINT £3, TAB(22); "from Grade"; I
46150 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3,
46160 PRINT £3, TAB(19); "Eventual Promotion to Grade"
46170 PRINT £3, "Total Length"; TAB(15); : FOR F = 1 TO 41: PRINT £3, "_"; : NEXT F
46180 PRINT £3,
46190 PRINT £3, "of Service"; : FOR F = 2 TO II
46200 PRINT £3, TAB((F - 2) * 8 + 16); F; : NEXT F
46210 PRINT £3,
46220 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3,
46230   FOR G = LMI(I) TO LMX(I) - 1
46240     PRINT £3, USING "££"; TAB(5); G;
46250     FOR J = I + 1 TO II
46260       PRINT £3, USING "££.££"; TAB((J - 1) * 8 + 7); EW(I, G, J);
46270     NEXT J
46280     PRINT £3,
46290   NEXT G
46300 FOR F = 1 TO 56: PRINT £3, "_"; : NEXT F: PRINT £3, : PRINT £3,
46320 RETURN
```

APPENDIX D - THE COMMANDS OF THE DECISION SUPPORT SYSTEM

To invoke the model builder and model optimiser, XPRESS-MP, a PC based MIP software, must be activated by typing:

CD\XPRESSMP

at the operating system command level, i.e. C:\> prompt. Then type:

MP-MODEL MOD5-9

at the prompt, C:\XPRESSMP>, to invoke the model builder, MP-MODEL, and to specify a problem name, in this case MOD5-9, for default specification for the remainder of the run. The model builder will display its prompt, the > character. Then type:

INPUT

at the prompt, >, to input model formulation from a file, MOD5-9.MOD, i.e. the MIP model (5-9). Note that the extension name MOD, which represents a model file, is automatically added to the problem name, MOD5-9. A matrix file, MOD5-9.MAT, for input to the model optimiser will be created by this model builder.

The optimiser, MP-OPT, is invoked by typing:

MP-OPT MOD5-9

at the prompt, C:\XPRESSMP>, where MOD5-9 is the problem name. Then at the prompt, >, type:

INPUT

to input the matrix file, MOD5-9.MAT, created by the model builder, into the optimiser. After inputting the matrix, then instruct the model optimiser to search for the minimum cost solution by typing:

MINIMISE

When an optimal solution to the linear relaxation has been found, the search for integer solutions is started by typing:

GLOBAL

Information on the process of searching for the integer solutions will be displayed. When the optimal solution has been found, save the solutions of decision variables and binary variables to ASCII files by typing:

ASCMSK='N???????'

TOASC MIPOUT

ASCMSK='X???????'

TOASC MIPX

respectively at each prompt, >. The command, ASCMSK, is used to control the variable names which are written to the ASCII file. Only vectors whose names match ASCMSK are written to the file specified by the TOASC command. From appendix C2 the decision variables and binary variables in the model file, MOD5-9.MOD, are initiated by N and X respectively. Their solutions are written to ASCII files, MIPOUT.ASC and MIPX.ASC, respectively. Note that when the solutions are infeasible it is unnecessary to save their output as ASCII files. Note also that the extension name, ASC, is automatically added to the file specified by the TOASC command. XPRESS-MP is left by typing:

QUIT

The bounds generator, overlapping ranges generator and results writer have been saved in a file, GENRATOR.BAS, a computer program in BASIC, which can be activated by typing:

QBASIC/RUN GENRATOR

at the prompt, C:\>. The screen will display the following message:

```
1. NARROW BOUNDS OF PROMOTION RATE RANGES        :

2. OVERLAP BOUNDS OF PROMOTION RATE RANGES       :

3. WRITE RESULTS                                 :

SELECT 1, 2, OR 3, PLEASE                        : ?
```

By selecting **1**, i.e. invoking the bounds generator, the screen will display the following message:

```
1. NARROW BOUNDS OF PROMOTION RATE RANGES        :
     enter reduction factor, Q                   : ?

2. OVERLAP BOUNDS OF PROMOTION RATE RANGES       :

3. WRITE RESULTS                                 :

SELECT 1, 2, OR 3, PLEASE                        : 1
```

The reduction factor Q, which is described in section 3.6, is used for reducing the promotion rate ranges width between successive iterations, where $Q \geq 1/J$, J is the number of promotion rate ranges and $H^{(k)}$ is the range width at iteration k, $k \geq 1$, i.e. at previous iteration. After entering the value of the reduction factor, the lower bounds and upper bounds of the promotion rate ranges in each grade are generated and then the prompt, C:\>, will be displayed.

By selecting **2**, i.e. invoking the overlapping ranges generator, the screen will display the following message:

```
1. NARROW BOUNDS OF PROMOTION RATE RANGES              :
2. OVERLAP BOUNDS OF PROMOTION RATE RANGES             :
    enter extension factor,Q*                          : ?
    enter reduction factor, Q                          :
    enter the number of overlapping ranges, J*         :
3. WRITE RESULTS                                       :

SELECT 1, 2, OR 3, PLEASE                              : 2
```

The extension factor Q^*, which is described in section 4.3, is used to extend an appropriate optimal range in order to increase the possibility of discovering the global optimal solution. This extended optimal range will be divided into J^* overlapping ranges. After entering the values of Q^*, Q and J^*, the lower bounds and upper bounds of the overlapping ranges in each grade, in which the upper bound of range j is an interior point of range j+1, are generated and the prompt, C:\>, will be displayed.

By selecting **3**, i.e. invoking the results writer, the probability of promotion from grade i to i+1 , i.e. using equation (6-1a), will be produced and stored in the file, RPB.OUT, and the solution of the MIP model will be written to the file, OUTPUT.TAB, in the form of tables, and to the files, MPN.GRF, MPNP.GRF, MPNT.GRF, MPNR.GRF, MPCOST.GRF and MPRP.GRF, in the forms for graphical presentations. After the selection, the system command level, C:\>, will be displayed. The probability of eventual promotion to a specified grade and the expected waiting time for the promotion then can be generated by typing

QBASIC/RUN PROB&WT

at the prompt, C:\>. These probabilities and waiting time are, therefore, stored in the file, EPRO&WT.TAB.

To invoke the graph generator, Harvard Graphics, a PC based graphics software, must be activated by typing:

CD\HG

at the prompt, C:\>. Then at the prompt, C:\HG>, type:

HG

The main menu of the Harvard Graphics will be displayed as below:

Create new chart	1
Enter/Edit chart	2
Draw/Annotate	3
Get/Save/Remove	4
Import/Export	5
Produce output	6
Slide show menu	7
Chartbook menu	8
Set up	9
Exit	E

Select Slide show menu by pressing **7**. The following menu will be displayed:

Create slide show	1
Edit slide show	2
Add ScreenShow effects	3
Display ScreenShow	4
Make practice cards	5
Select slide show	6

Chose Select slide show by pressing **6**. A table of filenames of the slide show will be displayed. Then highlight the show name, **MANPOWER.SHW**, and press **ENTER** key, the slide show menu described above will be displayed again.

Select Display ScreenShow by pressing **4**. The graphs of the manpower planning will be presented from screen to screen by pressing the **ENTER** key, or by using **LEFT ARROW** and **RIGHT ARROW** keys to view the preceding or subsequent charts, or by typing the **number** which has been assigned to the charts. To leave Harvard Graphics, press **ESC** key until the main menu is restored and then press **E** to exit the system.

To invoke the table generator, any word processor can be activated, or simply use DOS 5.0 operating system command, **EDIT**. Then select the filename, **OUTPUT.TAB**, to view results.